Slope Failure and Calculus

Content Area(s)/Course/Grade: AP Calculus	Unit: Techniques of Integration
Lesson Topic: Indefinite and Definite Integrals	Length of Lesson: 2-3 Days
Materials for Students: Calculator and writing utensil	Materials for Teacher: Grid plates and foam cutouts, guided worksheet/notes/presentation

Standard(s) Addressed:
Riemann Sums, Summation Notation, and Definite Integral Notation
The Fundamental Theorem of Calculus and Definite Integrals

Student Outcome(s):
I can express a definite integral in summation notation
I can evaluate indefinite integrals
I can evaluate definite integrals using the fundamental theorem of calculus

Context for Learning

This lesson will help show them how an infinite Reimann Sum will find the exact area under a curve and is set up with summation notation. The process to evaluate that will use the last lesson on antiderivatives which leads to solving indefinite integrals and definite integrals using FTC.

Instructional Delivery

Lesson notes: Day 1 of this lesson will be mostly teacher lead discussion and notes about the 3 student outcomes. Notes will begin with reflection on Reimann Sums and how to improve them. This should lead to summation notation approach of defining that area. Afterwards we would need to be able to evaluate that which is not possible by hand so the method of FTC will have to be used. While also developing FTC students will also learn about how exact geometric shapes could be used to find the area. We will practice several examples together as a class. Day 2 of the lesson will be a hands-on activity having the students apply these concepts to more examples of slope failure and rainfall data. After activity is completed students will be able to continue practicing the concept on their worksheet. Day 3 is only needed if activity went long or students need more practice with the material. (notes are attached)

Activity: Students will work in small groups and evaluate definite integrals based on slope failures and rainfall data. They will have to use a variety of techniques to find values in the questions. They will have to use Reimann Sums, Geometric formulas, FTC from a given function, and FTC from a set of points they turn into a function. (activity worksheet is attached)

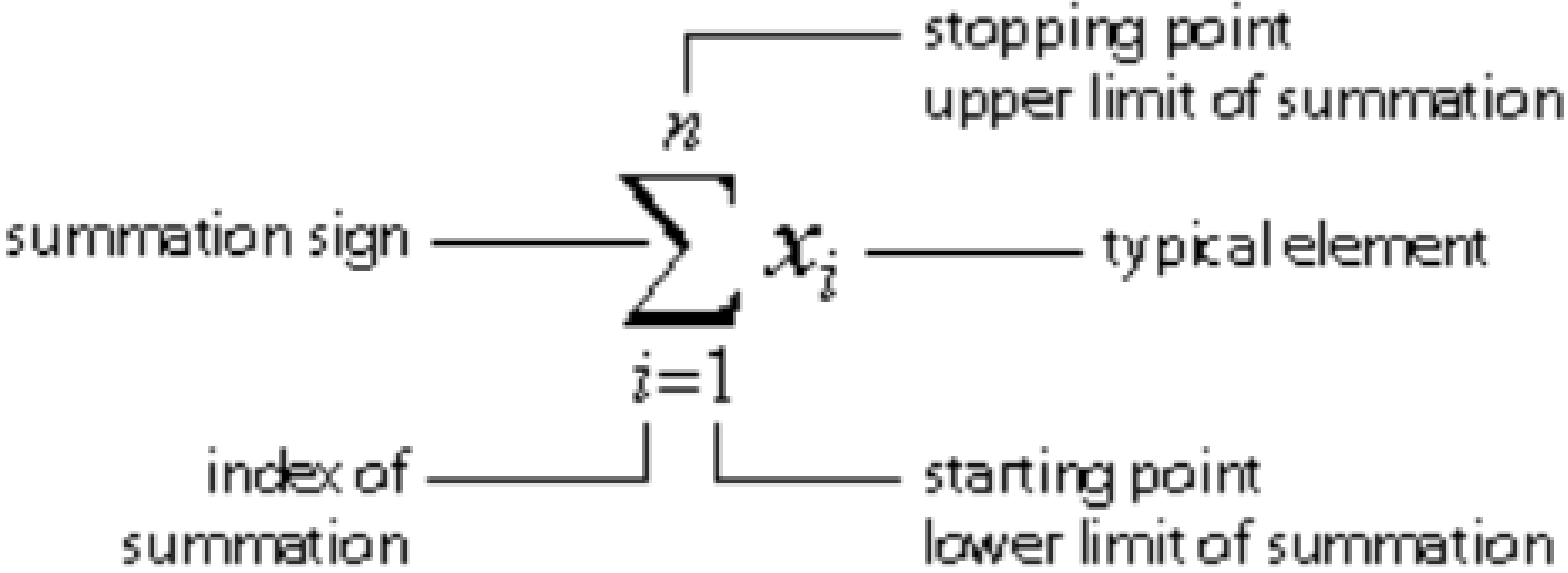
Assessment/Evaluation (Formative/Summative)

There will be an informal formative assessment in the form of their worksheet. Gather how they are doing by walking around to each student and observing them work. There will be a formal formative assessment the day after this lesson in the form of a mini quiz that will be one FTC problem and one question using exact geometric formulas to solve. (worksheet and mini quiz will be attached with this lesson plan)

Accommodations: Make sure everyone has a calculator for this worksheet and activity. Reduce the amount of question for those who need it. Walk around and help those students that need more help. Extra time for students that need it on the mini quiz.

Slope Failures and Calculus
Lesson 3 Summation Notation and FTC

Objectives: To be able to identify the parts of the formal definition of the integral and evaluate definite integrals exactly using fundamental theorem of calculus.



If we want to make our approximations better, what would we have to do?

How would we represent this symbolically

The definition of a definite integral has a similar look to that of the definition of a derivative.

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)dx$$

Like the derivative definition where the difference in x became 0 to find the instantaneous slope here the width is becoming 0 to find infinitely many rectangles to find the exact area.

Use the definition to represent the area under the function for any interval:

$f(x) = 5x^2 - 3$

Series and sequence formula and properties can be used to evaluate these. In some cases anti-derivatives must be evaluated this way. (in the case where we can not find the anti-derivative of the function)

The Fundamental Theorem of Calculus

If a function f is continuous on a closed interval [a,b] and F is an antiderivative of f on the interval [a,b], then $\int_a^b f(x)dx = F(b) - F(a)$

Evaluate each definite integral.

$\int_{-1}^{3} x^2 + 1dx$

$\int_1^4 2\sqrt{x}dx$

$$\int_0^{\frac{\pi}{4}} \sec^2 x dx$$

$$\int_0^2 |2x-3| dx$$

Find the area of the region bounded by the graphs of $y = 3x^2 - 6x + 6, x = 0, x = 3 \, and \, the \, x - axis$.

Slope Failures and Calculus Lesson 3 Fundamental Theorem of Calculus

Hand on Activity Day 3 Names in the group:_______________________

Slope failures happen across the world and often have terrible effects. We will learn more about slope failures as this unit progresses and how there is lots of calculus connected to the concept. In today activity your group you will need to work together, discuss possible techniques, and find the area of the foam sheets using the plastic sheet and the coordinate plane on it. The foam sheets now represent either land area or rainfall data. Today we will use the FTC to find the exact area of the foam sheet and explain the answer within the context. You will need to use a graphing calculator today for one of the questions to make a quadratic equation.

Foam sheet 1 (color)

Use the FTC to find the area of the foam sheet and explain what the area represents. (note any markings on the foam)

Foam sheet 2 (color)

Use the FTC to find the area of the foam sheet and explain what the area represents. (Note any markings on the foam and you will need to use your calculator to make a quadratic equation for this one. If you get stuck ask for help.)

SF and Calc Lesson 3 WS Name:_____________________________

1. The following limit is the definition for a definite integral. Explain the meaning of the parts of the limit (drawing on your knowledge about what a definite integral is and what it represents on a graph).

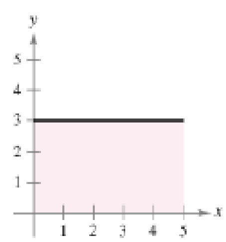
$$\lim_{\|\Delta\| \to \infty} \sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)dx$$

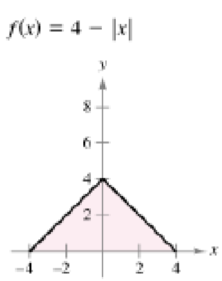
2. Write the limit as a definite integral on the interval [a, b].

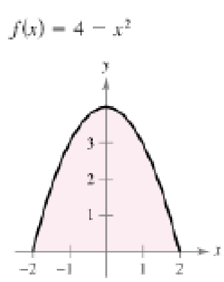
a. $\lim_{\|\Delta\| \to \infty} \sum_{i=1}^{n} (3c_i + 10)\Delta x_i$ on [-1, 5]

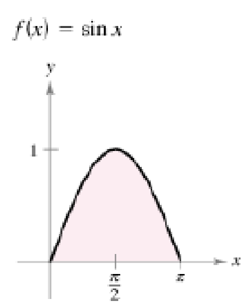
b. $\lim_{\|\Delta\| \to \infty} \sum_{i=1}^{n} (\frac{3}{c_i^2})\Delta x_i$ on [1, 3]

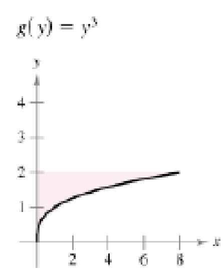
3. Set-up, but do not evaluate, the definite integral that yields the area of the region:

a. 

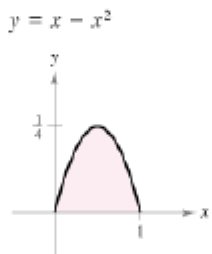
b. 

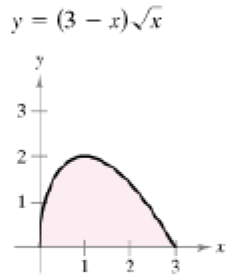
c. 

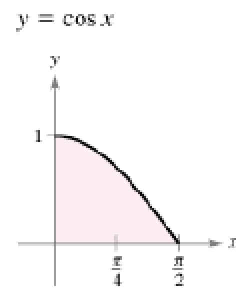
d. 

e. 

4. Determine the area of the given region:

a. 

b. 

c. 

5. Find the area bounded the graphs of the equations $y = 3x^2 + 1$, x = 0, x = 2, and y =0.

6. Evaluate the definite integral using algebraic methods (SHOW YOUR WORK)

a. $\int_1^3 (3x^2 + 5x - 4)dx$

b. $\int_0^1 (2t - 1)^2 dx$

c. $\int_1^8 (\sqrt{\frac{2}{x}})dx$

d. $\int_0^\pi (1 + \sin x)dx$

e. $\int_{-\pi/6}^{\pi/6} (\sec^2 x)dx$

f. $\int_{-\pi/2}^{\pi/2} (2t + \cos t)dt$

SF and Calc Lesson 3 Mini Quiz Name: ________________

Evaluate the definite integrals.

$\int_1^3 2x^3 - 3x^2 + 4x \, dx$

$\int_0^\pi -3\sin(x) \, dx$

SF and Calc Lesson 3 Mini Quiz Name: ________________

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