

# Slope Failure and Calculus

Content Area(s)/Course/Grade: AP Calculus

Unit: Techniques of Integration

Lesson Topic: Area between curves

Length of Lesson: 1 Day

Materials for Students: Calculator and writing utensil

Materials for Teacher: Grid plates and foam cutouts, guided worksheet/notes/presentation

Standard(s) Addressed:

Finding the Area Between the Curves Expressed as Functions of  $x$

Student Outcome(s):

I can find the area between two curves.

## Context for Learning

This lesson is often taught later in the class with volume problems, but it fits so naturally with the Slope Failure idea that a 1-day lesson on it now felt justified.

## Instructional Delivery

Lesson notes: There will be an activity first where they will have the chance to self-discover the method of finding the area between two curves. This activity will last about 20 minutes. The class will then have a couple notes on it and do a couple examples together this should last about 15 minutes. The last 10 minutes of the class will be time to work on the worksheet.

Activity: Students will work in small groups and evaluate definite integrals based on slope failures data. They will have to find the area before and after a slope failure and use those to find how much land moved during the slope failure. They will use FTC to evaluate it with a function given. (activity worksheet is attached)

## Assessment/Evaluation (Formative/Summative)

There will be an informal formative assessment in the form of their worksheet. Gather how they are doing by walking around to each student and observing them work. There will be a formal formative assessment the day after this lesson in the form of a mini quiz that will be one area between two curves question. (worksheet and mini quiz will be attached with this lesson plan)

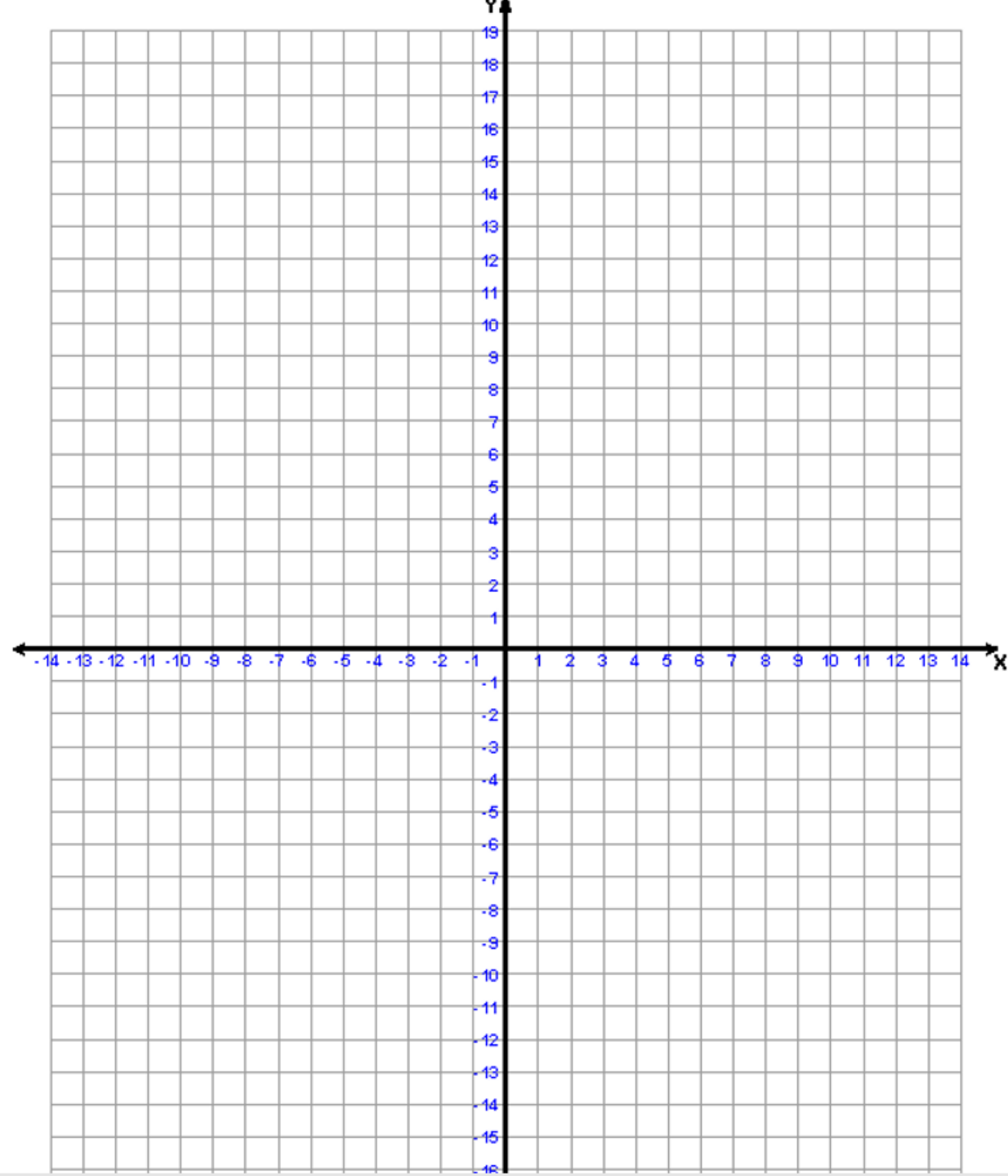
Accommodations: Make sure everyone has a calculator for this worksheet and activity. Reduce the amount of question for those who need it. Walk around and help those students that need more help. Extra time for students that need it on the mini quiz.

# Slope Failures and Calculus

## Lesson 5      Area Between Two Curves

Objectives: To be able to find the area between two curves.

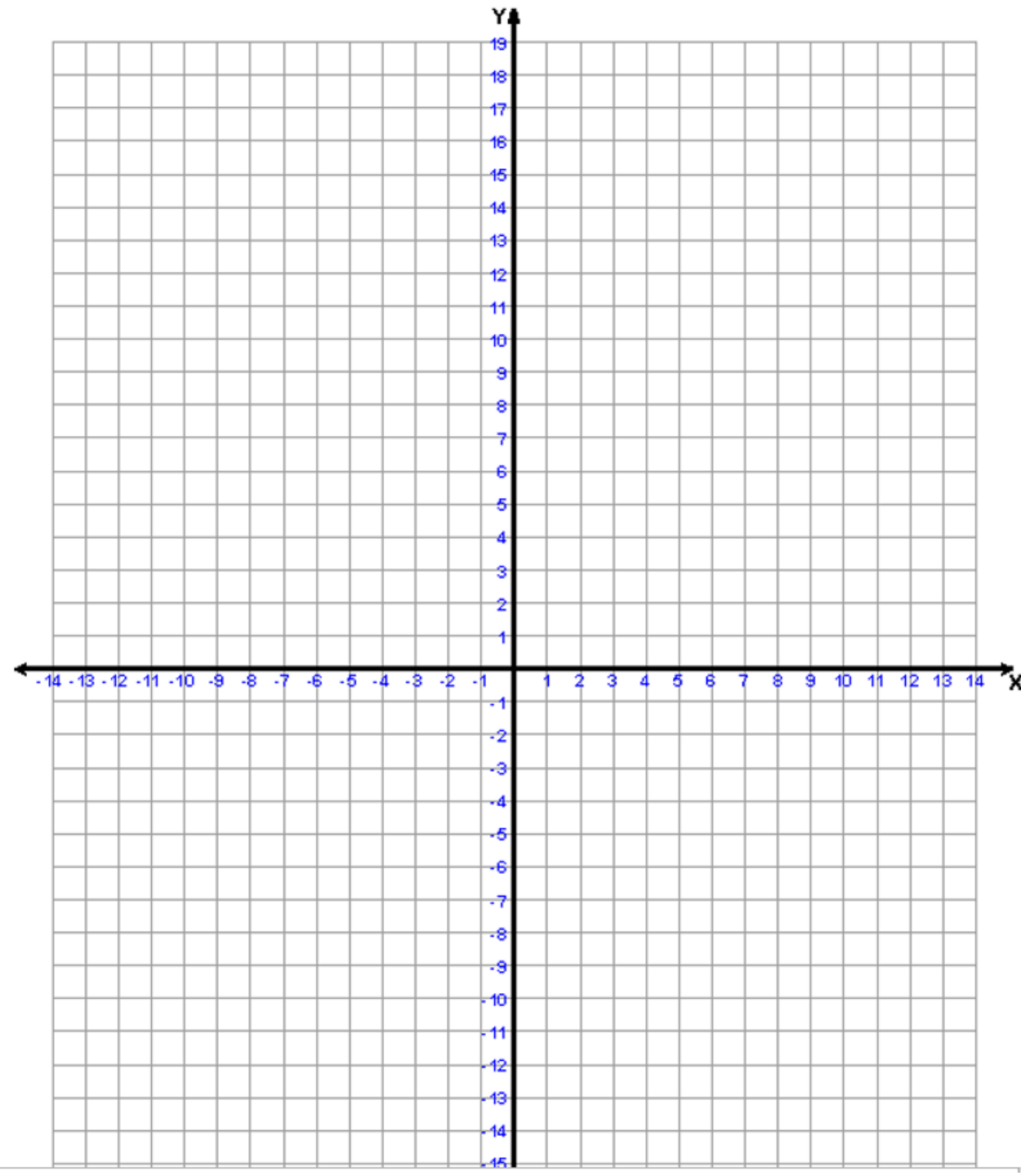
Sketch and find the area under the curve bounded by the x-axis and the lines  $x=-1$ ,  $x=2$ , and  $f(x) = 2x^3 + 3$



What would happen if you were not bounded by the x-axis?

What if we took the last example and bound it by the function  $g(x) = x-4$  instead of the x-axis what would this look like?

How would we set that up?



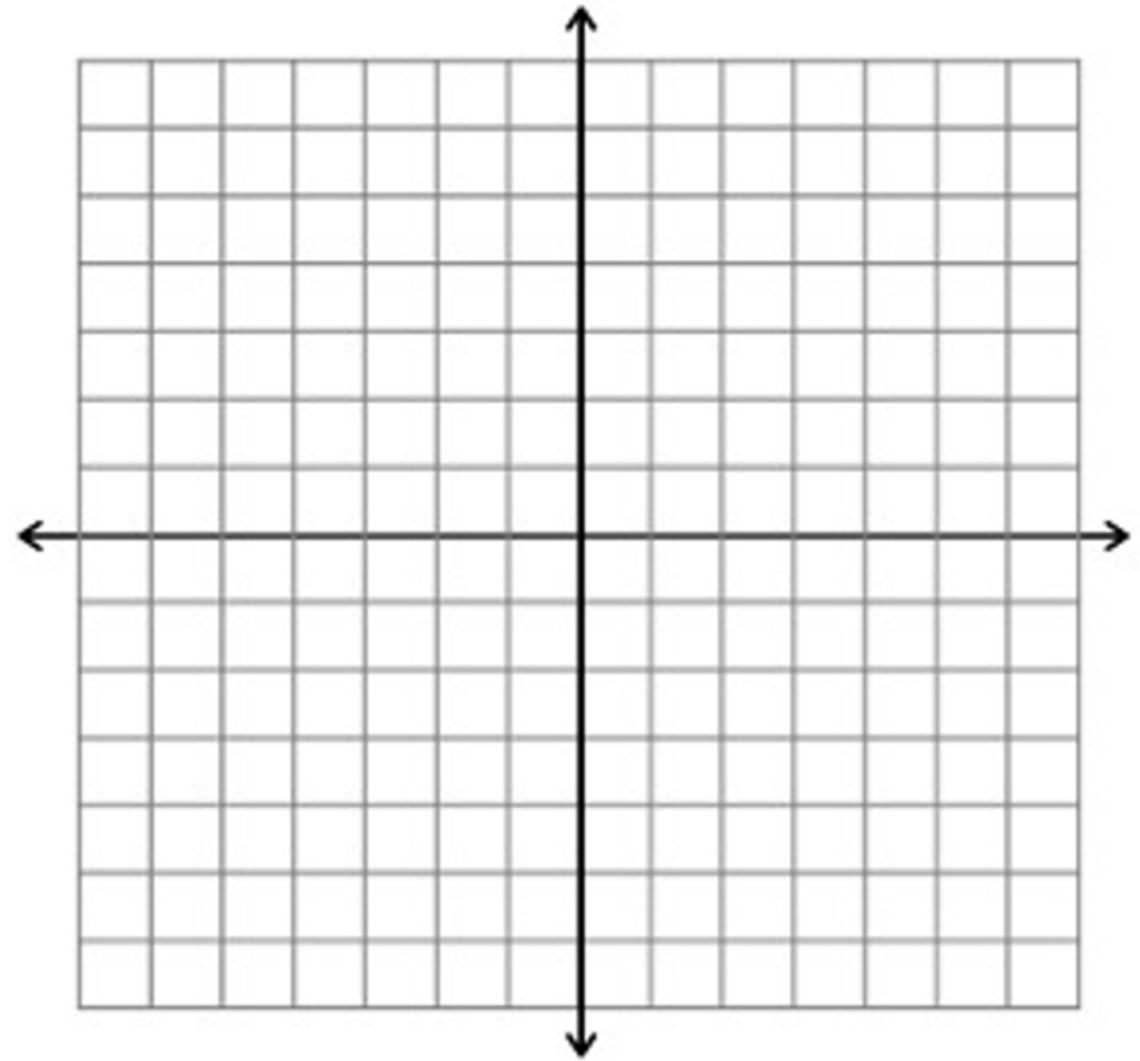
Find the area bounded by the curves  $y = 2 - x^2$  and  $y = x$

Steps

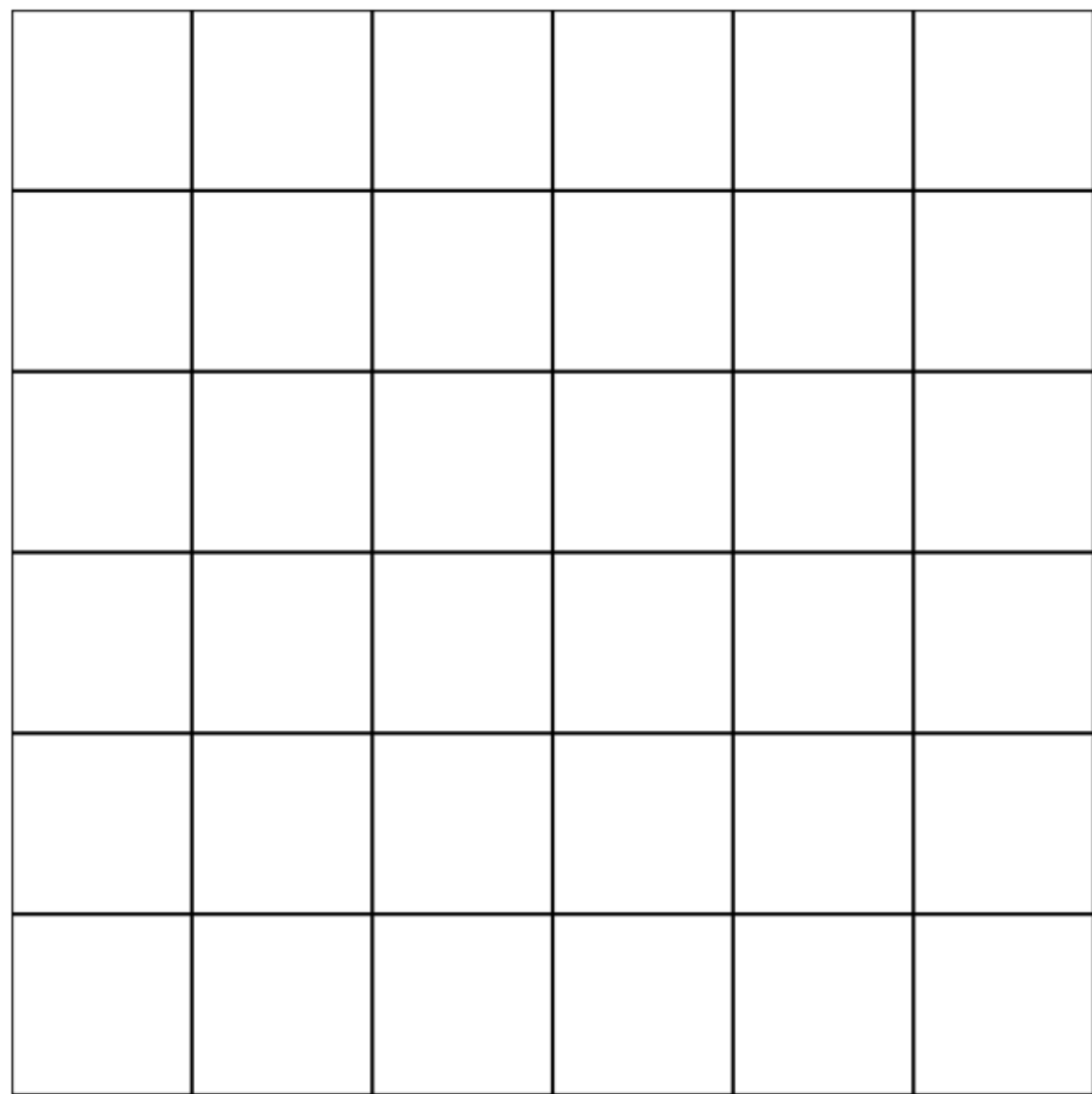
Graph the function and shade the area bounded by the curves

Find the intersection of the curves

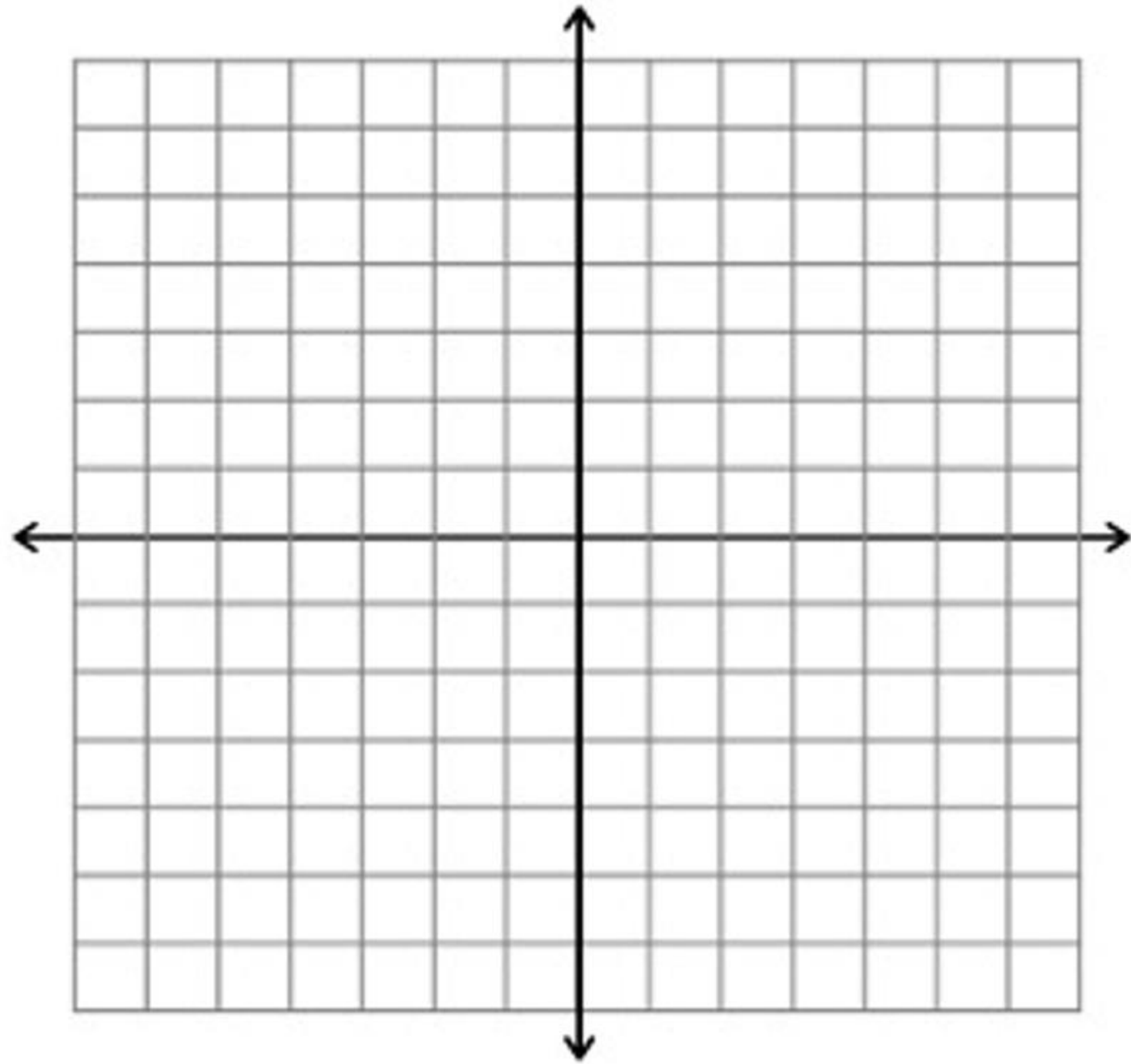
Integrate to find the area



Find the area bounded by the curves  $f(x) = (x - 1)^3$  and  $g(x) = x - 1$  between  $x=0$  and  $2$



Find the area under the curve bounded by  $y = \sqrt{x}$ ,  $y = x - 2$ , and the x-axis.



## Hand on Activity Lab 1

Names in the group: \_\_\_\_\_

Slope failures happen across the world and often have terrible effects. We will learn more about slope failures as this unit progresses and how there is lots of calculus connected to the concept. In today activity your group you will need to work together, discuss possible techniques, and find the area of the foam sheets using the plastic sheet and the coordinate plane on it. The foam sheets now represent land area of the same location but during two different dates. Today we will use the FTC to find the exact area of the foam sheet and explain the answer within the context. You will need to use a graphing calculator today for one of the questions to make a quadratic equation.

## Foam sheet 1 (color)

Use the FTC to find the area of the foam sheet and explain what the area represents. (Note any markings on the foam and you will need to use your calculator to make quadratic function.)

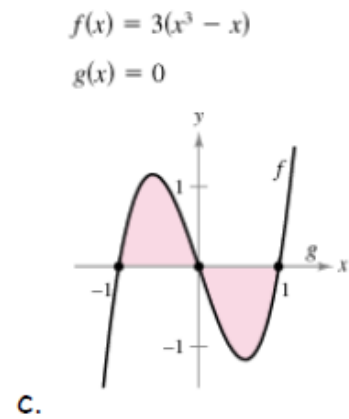
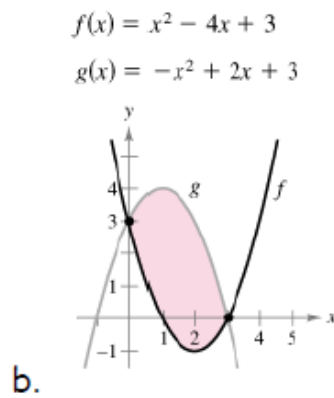
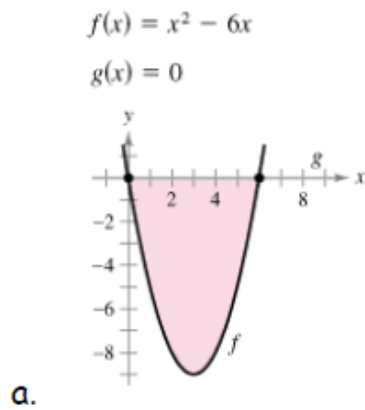
## Foam sheet 2 (color)

Use the FTC to find the area of the foam sheet and explain what the area represents. (Note any markings on the foam and you will need to use your calculator to make a power function. You will also need to use the calculator to evaluate this area)

Question: How much land moved during the slope failure? Is there a way you think you could have set this question up with out having to do the two prior parts?



## 1. Set-up the definite integral that gives the area of the region:



## 2. Sketch the region bounded by the graphs of the functions and find the area of the region.

a.)  $f(x) = x^2 - 1$  and  $g(x) = 1 - x^2$

b.)  $f(x) = x^2$  and  $g(x) = x + 2$

c.)  $f(x) = x^2$  and  $g(x) = x^3$

d.)  $f(x) = x^2$  and  $g(x) = 6 - x$

SF and Calc Lesson 5 Mini Quiz Name: \_\_\_\_\_

Find the area between the two curves,

$f(x) = -x^2 + 4x + 3$  and  $f(x) = -x^3 + 7x^2 - 10x + 5$   
on the interval  $[1,2]$

SF and Calc Lesson 5 Mini Quiz Name: \_\_\_\_\_

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