International Journal of Fracture **117:** L9–L14, 2002.

SCREW DISLOCATION INTERACTING WITH AN INTERFACIAL EDGE CRACK BETWEEN TWO BONDED DISSIMILAR PIEZOELECTRIC WEDGES

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Abstract: This letter is concerned with an interfacial edge crack in a piezoelectric bimaterial wedge interacting with a screw dislocation under antiplane mechanical and in-plane electric loading. In addition to a discontinuous electric potential across the slip plane, the dislocation is subjected to a line-force and a line-charge at the tip. The out-of-plane displacement and electric potentials are obtained in closed-form based on conformal mapping technique and the solution for the screw dislocation in an infinite piezoelectric bimaterial with a semi-infinite interfacial crack. The intensity factors (IFs) and energy release rate (EER) are obtained explicitly. These solutions can be used as a base for constructing solutions for arbitrary coupled antiplane mechanical and in-plane electrical loadings.

1. Introduction. Analysis of dislocations plays important role in fracture mechanics as dislocation solutions can be utilized as kernel functions for solving general crack problems, such as problems on cracks subjected to complicated loads, crack deflections, kinks, edge cracks, crack/crack interactions, crack/inclusion interactions, etc. A number of researchers contributed significantly to this subject, for example Thomson (1986), Suo (1989,1990), Atkinson and Eftaxiopoulos (1991), Weertman (1996), Huang and Kardomateas (2001), and Lee and Earmme (2000). With rapidly growing applications of piezoelectric ceramics in actuators, sensors, and transducers, studies of their fracture and failure mechanisms attract increasing attention in the literature. This includes analysis of defects in piezoelectric materials under coupled mechanical and electric loadings. As in elastic materials, dislocation solutions for cracked piezoelectric materials can serve as fundamental solutions to evaluate electroelastic fields in the presence of defects, such as dislocations, micro-cracks, cavities, inclusions, etc. Substantial progress was achieved in this area in the last two decades, in the general framework of linear piezoelectricity theory. Deeg (1980) analyzed piezoelectric solids with a dislocation, crack, or inclusion in coupled electromechanical fields. Pak (1990) studied the Peach-Koehler forces acting on a screw dislocation under external electromechanical forces. Lee et al (2000) and Chen et al (2002) investigated a screw dislocation interacting with a semi-infinite crack in a homogenous piezoelectric medium. Kwon and Lee (2002) studied a screw dislocation interacting with a finite crack in a homogenous piezoelectric material. Recently, Wu et al. (2002) obtained the general solutions for a screw dislocation interacting with an interface and collinear interfacial cracks in a piezoelectric bimaterial employing the superposition scheme used earlier by Thomson (1986) and Suo (1989, 1990).

It's well-known that piezoelectric ceramics are brittle and liable to cracking at the multitude of scales, from the scale of ferroelectric domains to devices (Suo et al., 1992). Most devices employ piezoelectric materials in structures containing bonds with other piezoelectric or elastic materials. Subjected to electromechanical loading, edge cracks are likely to occur at sharp corners and free edges of bonded dissimilar blocks, particularly near the corner of bonded dissimilar wedges where stress concentration exists even without cracks. In this letter, we consider a simple continuum model for a screw dislocation interacting with an interfacial edge crack between two bonded dissimilar piezoelectric wedges. Conformal mapping technique is utilized in conjunction with the previously obtained general solutions for piezoelectric bimaterials with collinear interfacial cracks (Wu et al., 2002). The out-of-plane displacement and in-plane electric potentials of the problem are obtained in closed-form. Furthermore, the intensity factors (IFs) and energy release rate (EER) of the edge crack are obtained explicitly.

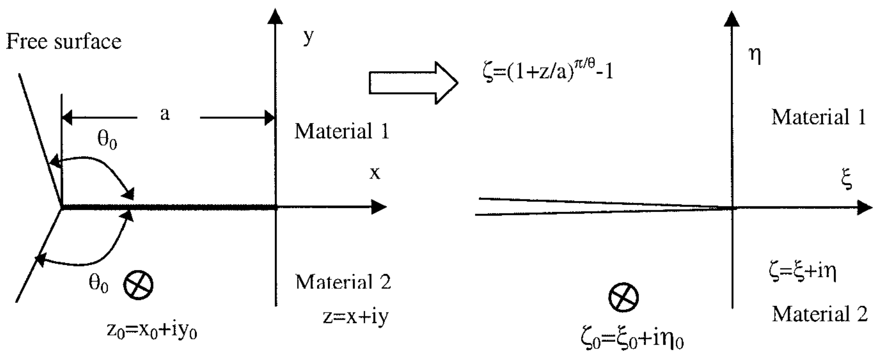
2. Problem formulation and solution procedure. Consider an interfacial edge crack of length a between two bonded dissimilar piezoelectric wedges, as shown in Fig. 1(a). Denote wedge angle as θ_0. Without loss of generality, suppose that a screw dislocation with Burgers vector b, a line-force p, a line-charge q, and an electric potential jump $\Delta\phi$ is located at z_0 in the lower wedge. The free-edge surfaces and the crack surfaces are treated as traction-free and electrically impermeable.

The piezoelectric media are considered to be transversely isotropic with hexagonal symmetry and the isotropic basal plane parallel to the xy-plane. The poling direction is assumed to be perpendicular to the xy-plane. By analogy with (Pak, 1990), the piezoelectric boundary value problem is formulated assuming non-zero out-of-plane mechanical displacement and in-plane electric field:

$$u_x = u_y = 0, \quad u_z = u_z(x, y), \tag{1}$$

$$E_x = E_x(x, y), \quad E_y = E_y(x, y), \quad E_z = 0. \tag{2}$$

In this case, the constitutive relations become



(a) z-plane (b) ζ-plane

Fig. 1 Interaction between a screw dislocation and an interfacial crack in a piezoelectric bimaterial

$$
\begin{aligned}
\sigma_{xz} &= c_{44}u_{z,x} + e_{15}\phi_{,x}, \quad D_x = e_{15}u_{z,x} - \varepsilon_{11}\phi_{,x}, \\
\sigma_{yz} &= c_{44}u_{z,y} + e_{15}\phi_{,y}, \quad D_y = e_{15}u_{z,y} - \varepsilon_{11}\phi_{,y},
\end{aligned}
\tag{3}
$$

where σ_{kz}, D_k (k=x,y), c_{44}, e_{15}, ε_{11} and ϕ are the stress tensor, electric displacement vector, elastic modulus at constant electric field, piezoelectric constants, and electric potential, respectively.

The electric field is given by

$$E_x = -\phi_{,x}, \quad E_y = -\phi_{,y}. \tag{4}$$

The governing equations are

$$\sigma_{xz,x} + \sigma_{yz,y} = 0, \qquad D_{x,x} + D_{y,y} = 0. \tag{5}$$

Substitution of (3) into (5) yields

$$c_{44}\nabla^2 u_z + e_{15}\nabla^2\phi = 0, \qquad e_{15}\nabla^2 u_z - \varepsilon_{11}\nabla^2\phi = 0. \tag{6}$$

The above equations can be reduced to two decoupled Laplace equations:

$$\nabla^2 u_z = 0, \quad \nabla^2\phi = 0. \tag{7}$$

As a result, the out-of-plane displacement and electric potentials can be expressed as imaginary parts of analytic functions U(z) and Φ(z)

$$u_z = \mathrm{Im}[\mathrm{U}(z)], \quad \phi = \mathrm{Im}[\Phi(z)], \tag{8}$$

where Im( ) denotes the imaginary part of an analytic function.

It is convenient to introduce the complex stress and electric displacement as:

$$
\begin{aligned}
\sigma_{zy} + i\sigma_{zx} &= c_{44}(u_{z,y} + iu_{z,x}) + e_{15}(\phi_{,y} + i\phi_{,\mathrm{x}}), \\
D_y + iD_x &= e_{15}(u_{z,y} + iu_{z,x}) - \varepsilon_{11}(\phi_{,y} + i\phi_{,\mathrm{x}}).
\end{aligned}
\tag{9}
$$

Using (8), (9) can be rewritten as

$$\sigma_{zy} + i\sigma_{zx} = c_{44}\,\mathrm{U}'(z) + e_{15}\Phi'(z),$$
$$D_y + iD_x = e_{15}\,\mathrm{U}'(z) - \varepsilon_{11}\Phi'(z). \tag{10}$$

Let us first consider a screw dislocation interacting with the semi-infinite interfacial crack in the infinite piezoelectric bimaterial, as shown in Fig.1 (b). The crack surfaces are assumed to be traction-free and electrically impermeable. The screw dislocation is located at $\zeta_0=\xi_0+i\eta_0$ in the lower half-plane and is characterized by a Burgers vector b, line-force p, line-charge q, and electric potential jump $\Delta\phi$. The complex potentials for this problem was obtained by Wu et al (2002) as

$$\begin{bmatrix} \mathrm{U}(\varsigma) \\ \Phi(\varsigma) \end{bmatrix}' = \begin{cases} 2(L_1+L_2)^{-1}L_2\mathbf{T}/(\varsigma-\varsigma_0) + 2L_1^{-1}\mathbf{h}(\varsigma), & \varsigma \in D_1, \\ (L_1+L_2)^{-1}(L_1-L_2)\overline{\mathbf{T}}/(\bar{\varsigma}-\bar{\varsigma}_0) + \mathbf{T}/(\varsigma-\varsigma_0) + 2L_2^{-1}\mathbf{h}(\varsigma), & \varsigma \in D_2, \end{cases} \tag{11}$$

where

$$\mathbf{h}(\varsigma) = H^{-1}/2\{\mathbf{T}/(\varsigma-\varsigma_0)[1-(\varsigma_0/\varsigma)^{1/2}] + \overline{\mathbf{T}}/(\varsigma-\bar{\varsigma}_0)[1-(\bar{\varsigma}_0/\varsigma)^{1/2}]\}. \tag{12}$$

Here $[\ ]'$ denotes the derivative of an analytic function with respect to $\zeta=\xi+i\eta$, D_1 and D_2 denote the upper and lower half-planes respectively, L_1 and L_2 represent the material matrices for the upper and lower half-planes, H is the bimaterial matrix, and $\mathbf{T}$ represents the screw dislocation quantity:

$$L = \begin{bmatrix} c_{44} & e_{15} \\ e_{15} & -\varepsilon_{11} \end{bmatrix}, \quad H = (L_1^{-1} + L_2^{-1}), \quad \mathbf{T} = \frac{1}{2\pi}\begin{bmatrix} b \\ \Delta\phi \end{bmatrix} + \frac{L_2^{-1}}{2\pi i}\begin{bmatrix} p \\ -q \end{bmatrix}. \tag{13}$$

In the xy-coordinate system (Fig. 1(a)), the IFs are defined as

$$\mathbf{K} = \sqrt{2\pi}\lim_{x\to 0} x^{1/2}[\mathbf{h}(x) + \overline{\mathbf{h}}(x)], \tag{14}$$

where $\mathbf{K} = \{\mathrm{K}_{\mathrm{III}}, \mathrm{K}_{\mathrm{D}}\}^{\mathrm{T}}$, and K_{D} is the electric intensity factor.

The traction and electric displacement at the interface a distance r ahead of the crack tip and the out-of-plane displacement and electric potential jumps a distance r behind of the crack tip can be expressed as

$$\mathbf{t}(r) = \mathbf{K}/\sqrt{2\pi r}, \quad \mathbf{d}(r) = H\mathbf{K}/\sqrt{2\pi r}, \tag{15}$$

with

$$\mathbf{t} = [\sigma_{yz}, D_y]^{\mathrm{T}}, \quad \mathbf{d} = [u_z(x,0+) - u_z(x,0-), \phi(x,0+) - \phi(x,0-)]^{\mathrm{T}}. \tag{16}$$

The ERR for a unit crack growth along the interface can be evaluated as

$$G = 1/(2\Delta)\int_0^{\Delta} \mathbf{t}^{\mathrm{T}}(\Delta - r)\mathbf{d}(r)dr = (1/4)\mathbf{K}^{\mathrm{T}}H\mathbf{K}. \tag{17}$$

Since the out-of-plane displacement and electric potentials satisfy decoupled Laplace equations (7), this problem can be solved explicitly by using the conformal mapping technique. Consider the conformal mapping

$$\zeta = (1 + z/a)^{\pi/\theta_0} - 1, \tag{18}$$

which maps the bonded piezoelectric wedges with an interfacial edge crack onto two bonded half-planes with a semi-infinite cut along the negative ξ-axis, as shown in Fig. 1.

Substitution of (18) into (11) and (12) leads to the potential solution for the edge-cracked bimaterial wedge. Its main contributor $\mathbf{h}(z)$ can be expressed as

$$\mathbf{h}(z) = \frac{\pi}{2\theta_0} H^{-1} \left\{ \frac{(z+a)^{\pi/\theta_0 - 1}\mathbf{T}}{(z+a)^{\pi/\theta_0} - (z_0+a)^{\pi/\theta_0}} \left[1 - \left(\frac{(z_0+a)^{\pi/\theta_0} - a^{\pi/\theta_0}}{(z+a)^{\pi/\theta_0} - a^{\pi/\theta_0}} \right)^{1/2} \right] \right.$$
$$\left. + \frac{(z+a)^{\pi/\theta_0 - 1}\overline{\mathbf{T}}}{(z+a)^{\pi/\theta_0} - (\bar{z}_0+a)^{\pi/\theta_0}} \left[1 - \left(\frac{(\bar{z}_0+a)^{\pi/\theta_0} - a^{\pi/\theta_0}}{(z+a)^{\pi/\theta_0} - a^{\pi/\theta_0}} \right)^{1/2} \right] \right\}. \quad (19)$$

Utilizing definition (14), the IFs can be evaluated as

$$\mathbf{K} = 2\pi\sqrt{2a^{\pi/\theta_0 - 1}} / \theta_0 H^{-1} \operatorname{Re}\{\mathbf{T}/[(z_0+a)^{\pi/\theta_0} - a^{\pi/\theta_0}]^{1/2}\}, \quad (20)$$

where $\mathbf{T}$ is the complex quantity of the screw dislocation defined in (13).

As shown in Fig. 2(a), consider two special cases of a line-force p and a line-charge q located at $z_0=-b$ (0<b<a) and $z_0=-a+he^{-i\theta_0}$, respectively. The loadings are located on the cut in the imaginary plane as shown in Fig. 2(b). Then, $\mathbf{T}$ can be rewritten as

$$\mathbf{T} = L_2^{-1}/(2\pi i)[p,-q]^{\mathrm{T}}. \quad (21)$$

Using (20), the respective IFs and ERR can be evaluated as

$$\mathbf{K} = \sqrt{\frac{2}{a\theta_0}} \sqrt{\frac{a^{\pi/\theta_0}}{a^{\pi/\theta_0} - (a-b)^{\pi/\theta_0}}} H^{-1} L_2^{-1} \begin{bmatrix} -p \\ q \end{bmatrix} \quad (22)$$

and

$$G = \frac{a^{\pi/\theta_0}}{2\theta_0 a[a^{\pi/\theta_0} - (a-b)^{\pi/\theta_0}]} \begin{bmatrix} -p \\ q \end{bmatrix}^{\mathrm{T}} L_2^{-1} H^{-1} L_2^{-1} \begin{bmatrix} -p \\ q \end{bmatrix} \quad (23)$$

for $z_0=-b$, and

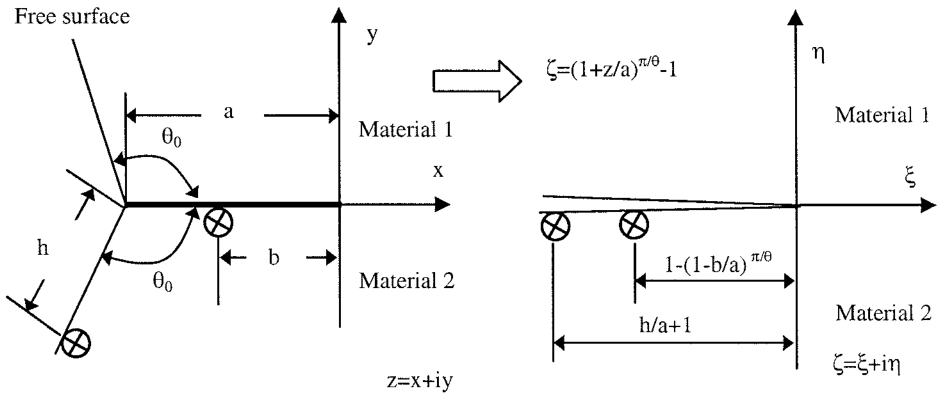
$$\mathbf{K} = \sqrt{\frac{2}{a\theta_0}} \sqrt{\frac{a^{\pi/\theta_0}}{a^{\pi/\theta_0} + h^{\pi/\theta_0}}} H^{-1} L_2^{-1} \begin{bmatrix} -p \\ q \end{bmatrix} \quad (24)$$

and

$$G = \frac{a^{\pi/\theta_0}}{2\theta_0 a(a^{\pi/\theta_0} + h^{\pi/\theta_0})} \begin{bmatrix} -p \\ q \end{bmatrix}^{\mathrm{T}} L_2^{-1} H^{-1} L_2^{-1} \begin{bmatrix} -p \\ q \end{bmatrix} \quad (25)$$

for $z_0=-a+he^{-i\theta_0}$.

Substitution of $\theta_0=\pi/2$, reduces the results (22)-(25) to those obtained by Li and Fan (2001) who used the method of dual integral equations.



(a) z-plane (b) ζ-plane

Fig. 2 A line-force and a line-charge located on the crack surface and the edge surface of a piezoelectric bimaterial wedge

Acknowledgement. Partial support of this work by the Air Force Office of Scientific Research and the Army Research Office is gratefully acknowledged.

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