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Screw Dislocation Interacting with Interfacial Edge-Cracks in Piezoelectric Bimaterial Strips

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Abstract

This paper is concerned with the interaction between an interfacial edge-crack and a screw dislocation under out-of-plane mechanical and in-plane electric loading in a piezoelectric bimaterial strip. In addition to a discontinuous electric potential across the slip plane, the dislocation is subjected to a line-force and a line-charge at the core. Under the framework of linear piezoelectricity, the out-of-plane displacement and in-plane electric potentials are constructed in closed-form by means of conformal mapping technique and the known solution for screw dislocation in cracked piezoelectric bimaterial. The intensity factors (IFs) and energy release rate (ERR) are derived explicitly.

Keywords: Screw dislocation; piezoelectric bimaterials; edge-crack; interfacial fracture; intensity factors (IFs); energy release rate (ERRs); strips; conformal mapping; butt joint

1. Introduction

The study of dislocations in solid materials plays important role in fracture mechanics since dislocation solutions can serve as Green's functions to construct the solutions for various crack configurations under arbitrary loadings. In the last two decades, a number of investigations have been contributed to this field [1-4]. With extensive applications of piezoelectric ceramics in actuators, sensors, and transducers *etc.*, the study of fracture and failure of piezoelectric materials has attracted numerous researchers' attention in recent years, especially the behaviors of various defects and inclusions under the coupled electric and mechanical loadings. For example, Deeg [5] first studied the response of piezoelectric solids with a dislocation, a crack, and an inclusion under coupled electromechanical fields, and Suo *et al.* [6] discussed the general solutions to collinear interfacial cracks in anisotropic piezoelectric bimaterials, *etc*. Detailed review of recent developments in fracture mechanics of piezoelectric materials can be found in the review paper by Zhang *et al*. [7]. In the study of screw dislocations in piezoelectric ceramics, Pak [8] first considered the Peach-Koehler forces acting on a screw dislocation under external electromechanical loadings. Lee *et al*. [9] and Chen *et al*. [10] dealt with a screw dislocation interacting with a semi-infinite crack in a transversely homogenous piezoelectric ceramics, and Kwon and Lee [11] further considered the similar case of a finite crack. Recently, by means of superposition, Soh [12,13], Wu *et al*. [14-16], and Liu *et al.* [17,18] obtained the fundamental solutions for screw dislocations in cracked piezoelectric bimaterial media with finite dimensions.

In reality, cracks more likely appear near free edges of bonded dissimilar blocks where stress

concentration exists even without cracks. In this work, we consider a simple continuum model of screw dislocation interacting with an interfacial edge-crack in a piezoelectric bimaterial strip. By means of conformal mapping technique and known fundamental solutions to screw dislocations in piezoelectric bimaterials [12,15], the out-of-plane displacement and in-plane electric potentials of the cracked strip are derived in closed-form. The intensity factors (IFs) and the energy release rate (ERR) of the edge-cracks are obtained explicitly.

2. Problem statement and solution procedure

As discussed in [8,9], the piezoelectric medium is supposed to be transversely isotropic, which has an isotropic basal plane parallel to *xy*-plane and a poling direction perpendicular to *xy*-plane. The boundary-value problem is treated as in the case of out-of-plane mechanical displacement and in-plane electric fields such that

$$u_x = u_y = 0,\ \ u_z = u_z(x,y), \tag{1}$$

$$E_x = E_x(x,y),\ \ E_y = E_y(x,y),\ \ E_z = 0. \tag{2}$$

In this case the constitutive relations reduce to

$$\begin{aligned} \sigma_{xz} &= c_{44}u_{z,x} + e_{15}\phi_{,x},\ \ D_x = e_{15}u_{z,x} - \varepsilon_{11}\phi_{,x}, \\ \sigma_{yz} &= c_{44}u_{z,y} + e_{15}\phi_{,y},\ \ D_y = e_{15}u_{z,y} - \varepsilon_{11}\phi_{,y}, \end{aligned} \tag{3}$$

where σ_{kz}, D_k (k=x, y), c_{44}, e_{15}, ε_{11} and ϕ are the stress tensor, electric displacement vector, elastic modulus at constant electric field, piezoelectric constants, and electric potential, respectively. The electric field is given by

$$E_x = -\phi_{,x},\ E_y = -\phi_{,y}. \tag{4}$$

The governing equations are

$$\sigma_{xz,x} + \sigma_{yz,y} = 0,\ D_{x,x} + D_{y,y} = 0. \tag{5}$$

Substitution of (3) into (5) leads to

$$\begin{aligned} c_{44}\nabla^2 u_z + e_{15}\nabla^2\phi = 0, \\ e_{15}\nabla^2 u_z - \varepsilon_{11}\nabla^2\phi = 0. \end{aligned} \tag{6}$$

The above equations may be reduced as

$$\nabla^2 u_z = 0,\ \nabla^2\phi = 0. \tag{7}$$

As a result, the out-of-plane displacement and in-plane electric potentials can be expressed as the imaginary parts of two analytic functions U(z) and Φ(z), *i.e.*

$$u_z = \mathrm{Im}[\mathrm{U}(z)],\ \phi = \mathrm{Im}[\Phi(z)], \tag{8}$$

where Im( ) denotes the imaginary part of an analytic function. It is convenient to introduce the complex stress and electric displacement:

$$\begin{aligned} \sigma_{zy} + i\sigma_{zx} &= c_{44}(u_{z,y} + iu_{z,x}) + e_{15}(\phi_{,y} + i\phi_{,x}), \\ D_y + iD_x &= e_{15}(u_{z,y} + iu_{z,x}) - \varepsilon_{11}(\phi_{,y} + i\phi_{,x}). \end{aligned} \tag{9}$$

Using (8), relation (9) may be rewritten as

$$\begin{aligned} \sigma_{zy} + i\sigma_{zx} &= c_{44}\,\mathrm{U}'(z) + e_{15}\Phi'(z), \\ D_y + iD_x &= e_{15}\,\mathrm{U}'(z) - \varepsilon_{11}\Phi'(z). \end{aligned} \tag{10}$$

Reserving the *xy*-coordinate system for the problem to be solved, we first determine the out-of-plane displacement and in-plane electric potentials for a screw dislocation located in a cracked bimaterial, as shown in *Fig. 1*. The crack surfaces are assumed to be traction-free and electric impermeable. The screw dislocation is assumed located at ζ_0 (ζ_0=ξ_0+iη_0) in the lower half-plane and characterized by Burgers vector *b*, line-force *p*, line-charge *q*, and electric potential jump $\Delta\phi$. The complex potentials for this problem have been determined [12,15] as

$$\begin{bmatrix} \mathrm{U}(\varsigma) \\ \Phi(\varsigma) \end{bmatrix}' = \begin{cases} 2(L_1+L_2)^{-1}L_2T/(\varsigma-\varsigma_0) + 2L_1^{-1}h(\varsigma) \\ \qquad \varsigma \in D_1\ \ i.e.(\eta > 0), \\ (L_1+L_2)^{-1}(L_1-L_2)\overline{T}/(\overline{\varsigma}-\overline{\varsigma}_0) \\ +T/(\varsigma-\varsigma_0) + 2L_2^{-1}h(\varsigma), \\ \qquad \varsigma \in D_2\ \ i.e.(\eta < 0), \end{cases} \tag{11}$$

where

$$\mathbf{h}(\varsigma) = B^{-1}/2\{\mathbf{T}/(\varsigma - \varsigma_0)[1-(\varsigma_0/\varsigma)^{1/2}] + \overline{\mathbf{T}}/(\varsigma - \bar{\varsigma}_0)1-(\bar{\varsigma}_0/\varsigma)^{1/2}\}. \quad (12)$$

Here the prime denotes the derivative of an analytic function with respect to $\zeta=\xi+i\eta$, D_1 and D_2 denote the half-planes above and below respectively, L_1 and L_2 represent the material matrices of the half-planes above and below, B is the bimaterial matrix, and $\mathbf{T}$ is the complex quantity of the screw dislocation:

$$L = \begin{bmatrix} c_{44} & e_{15} \\ e_{15} & -\varepsilon_{11} \end{bmatrix}, \; B = (L_1^{-1} + L_2^{-1}), \quad (13)$$
$$T = 1/(2\pi)[b, \Delta\phi]^T + L_2^{-1}/(2\pi i)[p, -q]^T.$$

In the xy-coordinate system as shown in *Figs. 2* and *3*, the IFs are defined as [15]

$$\mathbf{K} = \sqrt{2\pi}\lim_{x\to 0} x^{1/2}[\mathbf{h}(x) + \overline{\mathbf{h}}(x)], \quad (14)$$

where $\mathbf{K} = \{K_{III}, K_D\}^T$, and K_D is the electric intensity factor.

The traction and electric displacement at the interface a distance r ahead of the crack tip and the displacement and electric potential jumps a distance r behind of the crack tip are expressed as

$$\mathbf{t}(r)=\mathbf{K}/\sqrt{2\pi r}, \; \mathbf{d}(r)=B\mathbf{K}/\sqrt{2\pi r}, \quad (15)$$

with

$$\mathbf{t} = \begin{bmatrix} \sigma_{yz} \\ D_y \end{bmatrix}, \; \mathbf{d} = \begin{bmatrix} u_z(x,0+) \\ \phi(x,0+) \end{bmatrix} - \begin{bmatrix} u_z(x,0-) \\ \phi(x,0-) \end{bmatrix}. \quad (16)$$

The ERR for a unit crack growth along the interface can be evaluated as

$$G = \lim_{\Delta\to 0} 1/(2\Delta)\int_0^{\Delta} \mathbf{t}^{\mathrm{T}}(\Delta - r) \bullet \mathbf{d}(r)dr = (1/4)\mathbf{K}^{\mathrm{T}} B\mathbf{K}. \quad (17)$$

Due to the out-of-plane displacement and in-plane electric potentials satisfying two decoupled Laplace equations (7), conformal mapping technique is applicable for solving some interfacial edge-cracks in piezoelectric bimaterial strips.

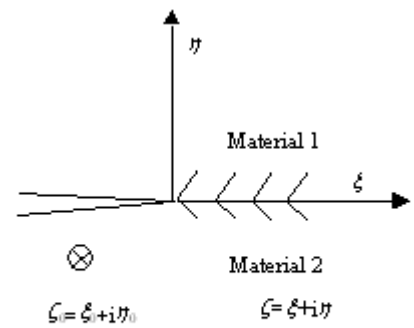

Fig. 1 screw dislocation in cracked piezoelectric bimaterial

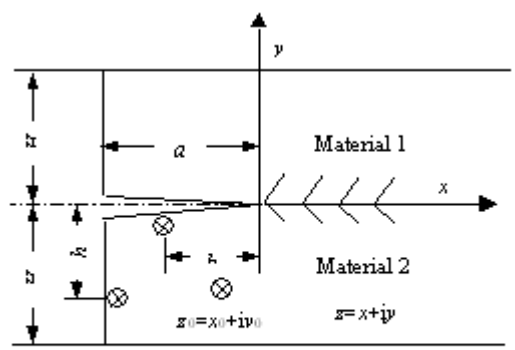

Fig. 2 Screw dislocation in edge-cracked piezoelectric strip

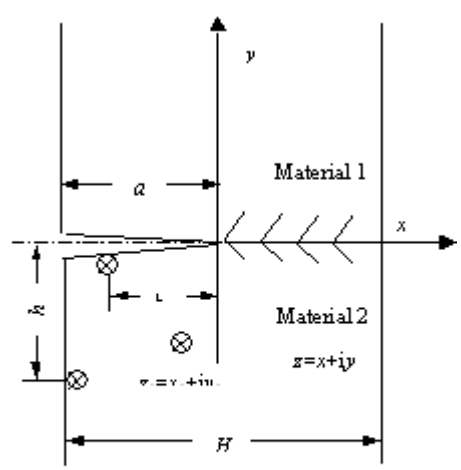

Fig. 3 Screw dislocation in edge-crack piezoelectric butt joint

3. Edge-cracked piezoelectric bimaterial strips

Consider a screw dislocation interacting with an interfacial edge-crack between two side-bonded dissimilar piezoelectric strips of equal width, as shown in *Fig. 2*, in which a and H denote the crack length and the strip width, respectively. The crack surfaces and the edge surfaces are assumed to be traction-free and electric impermeable. The screw dislocation is located at z_0 ($z_0=x_0+iy_0$) in the lower strip.

Introduce the conformal mapping

$$\zeta = f^2(z+a)/f^2(a)-1 \tag{18}$$

with

$$f(z)=\sinh[\pi z/(2H)], \tag{19}$$

which maps the bonded piezoelectric strips (*Fig. 2*) with an interfacial edge-crack onto two bonded half-planes with a semi-infinite cut along the negative ξ-axis, as shown in *Fig. 1*. Substitution of (18) and (19) into (11) and (12) leads to the potential of the strip. Its main contributor (12) may be expressed as

$$\begin{aligned} h_1(\mathrm{z}) &= \pi B^{-1}/(4H)\sinh[\pi(z+a)/H] \\ &\times\Big\{T/[f^2(\mathrm{z}+a)-f^2(\mathrm{z}_0+a)] \\ &\times\left[1-\left(\frac{f^2(z_0+a)-f^2(a)}{f^2(z+a)-f^2(a)}\right)^{1/2}\right] \\ &+\overline{T}/[f^2(\mathrm{z}+a)-\overline{f^2(\mathrm{z}_0+a)}] \\ &\times\left[1-\left(\frac{\overline{f^2(z_0+a)}-f^2(a)}{f^2(z+a)-f^2(a)}\right)^{1/2}\right]\Big\}. \end{aligned} \tag{20}$$

By the definition (14), the IFs are evaluated as

$$\begin{aligned} \mathbf{K} &= 2\sqrt{2}\pi B^{-1}/\sqrt{\pi a}\sqrt{\pi a/H\sinh(\pi a/H)} \\ &\times \mathrm{Re}\left\{-i\,\mathbf{T}\Big/\sqrt{\cosh(\pi a/H)-\cosh[\pi(z_0+a)/H]}\right\}, \end{aligned} \tag{21}$$

where $\mathbf{T}$ is the complex quantity of the screw dislocation defined in (13). By letting $H\to\infty$ in (21), we obtain the IFs for an interfacial edge-crack between two bonded dissimilar piezoelectric quarter-planes as

$$\mathbf{K} = 4\pi B^{-1}/\sqrt{\pi a}\,\mathrm{Re}\left\{-i\,\mathbf{T}\mathrm{a}/\sqrt{a^2-(a+z_0)^2}\right\}. \tag{22}$$

Furthermore, by letting $a\to\infty$ in (21), we get the IFs for a semi-infinite crack between two infinite side-bonded piezoelectric strips as

$$\mathbf{K} = 2\sqrt{2}\pi B^{-1}/\sqrt{H}\,\mathrm{Re}\left[-i\,\mathbf{T}\Big/\sqrt{1-\exp(\pi z_0/H)}\right] \tag{23}$$

As shown in *Fig. 2*, here we further consider two special loading cases of a line-force P_1 and a line-charge Q_1 located at $z_0=-b-0i$ (0<b<a) and a line-force P_2 and a line-charge Q_2 located at $z_0=-a-ih$ ($h<H$), respectively. Then, $\mathbf{T}$ may be rewritten as

$$\mathbf{T}_1 = L_2^{-1}/(2\pi i)[P_1,-Q_1]^T,\ \mathbf{T}_2 = L_2^{-1}/(2\pi i)[P_2,-Q_2]^T. \tag{24}$$

With the aid of (17) and (21), the IFs and ERR are evaluated respectively as

$$\begin{aligned} \mathbf{K} &= \sqrt{2}B^{-1}L_2^{-1}/\sqrt{\pi a}\sqrt{\pi a/H\sinh(\pi a/H)} \\ &\div\sqrt{\cosh[\pi a/(2H)]-\cosh[\pi(a-b)/H]}[-P_1,Q_1]^T, \end{aligned} \tag{25}$$

and

$$\begin{aligned} \mathrm{G} &= \frac{1}{2H}\frac{\sinh(\pi a/H)}{\cosh[\pi a/(2H)]-\cosh[\pi(a-b)/(2H)]} \\ &\times[-P_1,Q_1]L_2^{-1}B^{-1}L_2^{-1}[-P_1,Q_1]^T \end{aligned} \tag{26}$$

for $z_0=-b-0i$, and

$$\begin{aligned} \mathbf{K} &= \sqrt{2}B^{-1}L_2^{-1}/\sqrt{\pi a}\sqrt{\frac{\pi a/H\sinh(\pi a/H)}{\cosh(\pi a/H)-\cos(\pi h/H)}} \\ &\times[-P_2,Q_2]^T, \end{aligned} \tag{27}$$

and

$$\begin{aligned} \mathrm{G} &= \frac{1}{2H}\frac{\sinh(\pi a/H)}{\cosh(\pi a/H)-\cos(\pi h/H)} \\ &\times[-P_2,Q_2]L_2^{-1}B^{-1}L_2^{-1}[-P_2,Q_2]^T, \end{aligned} \tag{28}$$

for $z_0=-a-ih$.

If letting $H\to\infty$, results (25)-(28) cover those given by Li and Fan [19], who used the method of dual integral equations.

Furthermore, integration of (25) with respect to b in the interval [0, a] leads to the solutions for a Griffith crack embedded at the mid-plane of a piezoelectric layer under out-of-plane mechanical and in-plane electric loading as

those discussed by Li and Duan [20,21] and Li [22].

4. Interfacial edge-crack in piezoelectric bimaterial butt joint

Now let us consider the second case of a screw dislocation interacting with an interfacial edge-crack in a piezoelectric bimaterial butt joint, as shown in *Fig. 3*. Here a and H denote the crack length and the strip width, respectively. As aforementioned, the crack surfaces and the edge surfaces are assumed to be traction-free and electric impermeable. The screw dislocation is supposed located at z_0 ($z_0=x_0+iy_0$) in the lower strip. In this case, we choose the conformal mapping

$$\zeta = g^2(z+a)/g^2(a) - 1 \tag{29}$$

with

$$g(z) = \tan[\pi z/(2H)], \tag{30}$$

which maps the cracked butt joint (*Fig. 3*) onto two bonded half-planes with a semi-infinite cut along the negative ξ-axis, as shown in *Fig. 1*. Substituting (29) and (30) into (11) and (12), we obtain the potential for this problem. Its main contributor (12) may be expressed in terms of

$$\begin{aligned}\mathbf{h}_2(z) &= \pi B^{-1}/(2H)\sec^2[\pi(z+a)/(2H)] \\ &\times \Big\{ g(z+a)\mathbf{T}/[g^2(z+a)\text{-}g^2(z_0+a)] \\ &\times \left[1-\left(\frac{g^2(z_0+a)-g^2(a)}{g^2(z+a)-g^2(a)}\right)^{1/2}\right] \\ &+ g(z+a)\overline{\mathbf{T}}/[g^2(z+a)\text{ - }\overline{g^2(z_0+a)}] \\ &\times \left[1-\left(\frac{\overline{g^2(z_0+a)}-g^2(a)}{g^2(z+a)-g^2(a)}\right)^{1/2}\right]\Big\}.\end{aligned} \tag{31}$$

With the help of definition (14), we obtain the IFs as

$$\begin{aligned}\mathbf{K} &= 4\pi B^{-1}/\sqrt{\pi a}\sec[\pi a/(2H)] \\ &\times \mathrm{Re}\Big\{\text{- } i\mathbf{T}\sqrt{\pi a/(2H)\tan[\pi a/(2H)]} \\ &\div \sqrt{\tan^2[\pi a/(2H)]-\tan^2[\pi(z_0+a)/(2H)]}\Big\}\end{aligned} \tag{32}$$

By letting $H\to\infty$, (32) covers the IFs for an interfacial edge-crack between two bonded dissimilar piezoelectric quarter-planes as (22). Furthermore, by letting $a\to\infty$ and simultaneously keeping $c=(H-a)$ constant, we have the IFs for a semi-infinite interfacial crack heading towards a free surface such that

$$\mathbf{K} = 4\pi B^{-1}/\sqrt{\pi c}\,\mathrm{Re}\left[\mathbf{T}(c-z_0)/\sqrt{z_0(2c-z_0)}\right] \tag{33}$$

Similar to Section 3, here we consider the same loadings shown in *Fig. 3*. With the aid of (24) and (32), the IFs and ERR are evaluated respectively as

$$\begin{aligned}\mathbf{K} &= 2/\sqrt{\pi a}\sec[\pi a/(2H)] \\ &\times\sqrt{\frac{\pi a/(2H)\tan[\pi a/(2H)]}{\tan^2[\pi a/(2H)]-\tan^2[\pi(a-b)/(2H)]}} \\ &\times B^{-1}L_2^{-1}[-P_1,Q_1]^T\end{aligned} \tag{34}$$

and

$$\begin{aligned}\mathrm{G} &= \frac{1}{2H}\frac{\tan[\pi a/(2H)]\sec^2[\pi a/(2H)]}{\tan^2[\pi a/(2H)]-\tan^2[\pi(a-b)/(2H)]} \\ &\times[-P_1,Q_1]L_2^{-1}B^{-1}L_2^{-1}[-P_1,Q_1]^T\end{aligned} \tag{35}$$

for $z_0=-b-0i$, and

$$\begin{aligned}\mathbf{K} &= 2/\sqrt{\pi a}\sec[\pi a/(2H)] \\ &\times\sqrt{\frac{\pi a/(2H)\tan[\pi a/(2H)]}{\tan^2[\pi a/2H]+\tanh^2[\pi h/(2H)]}} \\ &\times B^{-1}L_2^{-1}[-P_2,Q_2]^T\end{aligned} \tag{36}$$

and

$$\begin{aligned}\mathrm{G} &= \frac{1}{2H}\frac{\tan[\pi a/(2H)]\sec^2[\pi a/(2H)]}{\tan^2[\pi a/(2H)]+\tanh^2[\pi h/(2H)])} \\ &\times[-P_2,Q_2]L_2^{-1}B^{-1}L_2^{-1}[-P_2,Q_2]^T\end{aligned} \tag{37}$$

for $z_0=-a-ih$. By letting $H\to\infty$, results (34)-(37) again return to those given by Li and Fan [19] and Wu *et al*. [15].

Since the general potentials for the above crack configurations have been obtained, the entire electroelastic fields of the strips and the forces acting at the screw dislocation can be extracted explicitly based on the method discussed by Pak [8].

5. Concluding remarks

Explicit solutions to screw dislocation interacting with interfacial edge-crack between two bonded dissimilar piezoelectric strips have been determined in this work. Conformal mapping technique has been shown to be a powerful tool in solving some interfacial edge-cracks in piezoelectric strips. Closed-form solutions (21) and (32) can be employed as a useful theoretical base for the assessment of numerical analyses, especially for estimating the effect of a/H ratio on the IFs and ERR of cracks in bonded piezoelectric structures. Furthermore, these explicit solutions can serve as Green's functions to construct the IFs and ERRs of piezoelectric strips with multiple cracks under arbitrary out-of-plane mechanical and in-plane electric loadings.

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