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# Wrinkling of a charged elastic film on a viscous layer

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**Abstract** A thin metallic film deposited on a compliant polymeric substrate begins to wrinkle under compression induced in curing process and afterwards cooling of the system. The wrinkle mode depends upon the thin film elasticity, thickness, compressive strain, as well as mechanical properties of the compliant substrate. This paper presents a simple model to study the modulation of the wrinkle mode of thin metallic films bonded on viscous layers in external electric field. During the procedure, linear perturbation analysis was performed for determining the characteristic relation that governs the evolution of the plane-strain wrinkle of the thin films under varying conditions, i.e., the maximally unstable wrinkle mode as a function of the film surface charge, film elasticity and thickness, misfit strain, as well as thickness and viscosity of the viscous layer. It shows that, in proper electric field, thin film may wrinkle subjected to either compression or tension. Therefore, external electric field can be employed to modulate the wrinkle mode of thin films. The present results can be used as the theoretical basis for wrinkling analysis and mode modulation in surface metallic coatings, drying adhesives and paints, and microelectromechanical systems (MEMS), etc.

**Keywords** Thin films · Wrinkling · Viscous flow · Pattern modulation · Applied mechanics

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## 1 Introduction

Wrinkling as a common surface instability phenomenon occurs in nature and engineered film systems, which is resulted from the redistribution of strain energies between the thin hard surface layers and the compliant substrates [1–3,9,17,34; and refs. therein]. In such film systems, strain mismatch usually builds up during the natural or manufacturing process such as aging, cooling, and solvent evaporation, annealing, and sol-gel transition of the compliant polymeric substrates, etc. For a thin elastic film deposited on a compliant substrate with sufficient interface strength, compressive misfit strains in the film generally evoke its elastic wrinkling under small perturb to reduce the strain energy stored in the system [4,5,13–16,21,26]. On the other hand, strain mismatch in a thin film system with weak interface strength may result in buckling delamination [10,18]. For thin elastic film wrinkling on compliant substrate, recent theoretical investigations have made great progress in understanding the wrinkling mechanism and relevant characteristic relation that governs the wrinkle growth. By using linear perturbation, Kirchhoff plate theory, and solution to a viscous layer with forced surface perturb, Sridhar et al. [26] first obtained the onset of maximally unstable mode of a compressed film wrinkling on a linearly viscous substrate as a function of the misfit strain and the thickness and viscosity of the viscous substrate layer. It was found that the onset of the film wrinkling on a glass layer is the same as that for a compressively stressed free-standing film though the maximally unstable wrinkle wavelength grows with the increase of the glass layer thickness. Furthermore, by means of nonlinear von Karman plate theory and Reynolds lubrication theory, Liang et al. [21] and Huang and Suo [13] refined the above study and considered the relaxation of compressed finite islands and thin films on viscous layers. In the limiting cases, results obtained in their models are in a good agreement with those predicted by Sridhar et al. [26]. Furthermore, Huang and Suo [14] and Huang [11] extended their earlier approaches to study thin film wrinkling on viscous layer with arbitrary thickness and on viscoelastic half-space. The general conclusions are similar to those of their earlier works. Moreover, to study the 2D patterns of thin films wrinkling on compliant substrates, Huang et al. [15] developed a concise 2D spectral scheme to simulate the wrinkle evolution in a numerical manner. The nonlinear effect due to finite magnitudes of the wrinkle modes was further taken into account in their later investigation [16]. As a result, the wrinkle patterns obtained in the 2D spectral simulation match those observed in experiments very well [1,4,17]. The above studies can be used to explore the wrinkling mechanisms and related characterization and suppression in thin films. Nevertheless, in view of application, it is desirable if wrinkling of thin films can be developed in a controllable manner.

Among others, external electric field would be one of the best candidates used for wrinkling modulation. In recent pioneering study in nanolithography, researchers have formulated highly ordered surface patterns through surface instablization of thin compliant dielectric polymeric layers in electric field [6,7,24,25]. Relevant theories regarding conditions of linear and nonlinear surface stabilities of the thin dielectric polymeric layers have been developed recently [22,23,29,31,32]. Full-field simulations of pattern evolution in thin dielectric polymeric layers in electric field have also been conducted by Wu and Russel [30] and Wu et al. [31, 32] using a 2D electrohydrodynamic viscous layer model, and by Kim and Lu [19] using a 3D spectral electrohydrodynamic phase-separation model. Furthermore, Huang [12] recently developed a simple model to further examine the instability criterion of a charged thin conductive elastic film bonded on a compliant dielectric layer; and Wu and Dzenis [33] considered the electrohydrodynamic instability of an ultrathin conductive viscous layer in electric field. In the latter case, the electrostatic surface traction induced by electric field was determined by the Maxwell stress tensor [20]. These investigations have indicated that electric field did play a vital role in wrinkle evolution of thin films and can be utilized for the purpose of wrinkling modulation.

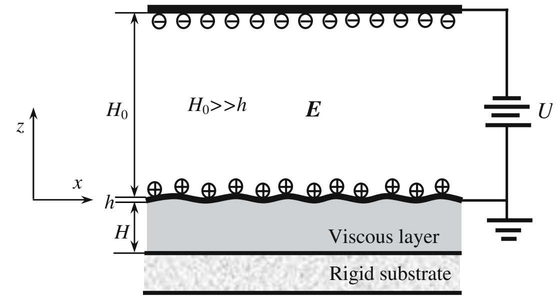


**Fig. 1** Wrinkling of a conductive elastic film on a viscous layer in external electric field

Therefore, it has pragmatic value to consider the mode modulation of thin metallic film wrinkling on viscous layer in external electric field. A simple plane-strain wrinkling model is shown in Fig. 1, where the upper conductive flat plate is adopted to sustain a constant far electric field. Without loss of generality, the thin conductive film is assumed being grounded. For thin film subjected to electric field, film tractions induced by surface charges vary with the density of net charges at film surfaces [20]. The charge density is a function of film profile that obeys the electrostatic law.

In an attempt to investigate the effect of surface charges on the wrinkle mode of a thin metallic film bonded on a viscous layer in electric field, in this study, we perform a linear perturbation analysis of a charged film-substrate system (see Fig.1). The characteristic relation that governs the plane-strain wrinkling is determined, e.g., the critical wrinkle mode number and the wrinkle growth rate as functions of the strength of electric field, film elasticity, viscosity of the substrate layer, and system geometries, respectively. Variation of the wrinkle growth rate versus the wrinkle mode number is plotted under varying condition. Limiting cases are examined and compared with data available in literature.

## 2 Problem formulation and solution

Consider a charged thin metallic film on a viscous layer that is located on a flat rigid substrate, as shown in Fig. 1. The system may be modeled as the bending of a compliant beam under the coupling action of the axial stress, lateral nontrivial flow, and electrostatic pressure induced by small surface perturb. The interface between the thin film and the viscous layer is assumed ideal, i.e., displacements and stresses are continuous across the interface. In the following, the inertia effect of the thin film and the viscous layer is ignored. Furthermore, since we are interested in the wrinkle initiation in the film system asymptotically close to its equilibrium state, the trivial constant forces such as those induced by the film and viscous layer gravities and the air pressure are safely neglected. The thin metallic film is considered as conductive and linearly elastic, and the viscous layer is regarded as incompressible newtonian flow.

### 2.1 Electrostatic tractions of a wrinkled conductive film in constant electric field

For unperturbed film, the potential of the electric field above the film may be expressed as

$$\phi = -4\pi\sigma_0 z, \tag{1}$$

where $\sigma_0$ is the charge density at the film surface which varies with the change of the plate distance $H_0$ (see Fig. 1), and $z$ is the vertical coordinate from the flat film surface. For a small perturb of the thin film:

$$w = A(t)\sin(kx) \tag{2}$$

with $A(t)$ and $k$ are, respectively, the perturb amplitude and the perturb wave number, the potential of the electrostatic field above the thin film can be expressed as [20]

$$\phi = -4\pi\sigma_0 z + \phi_1. \tag{3}$$

Here $\phi_1$ is a small correction satisfying $\Delta\phi_1 = 0$ and vanishing for $z \to \infty$. Thus, $\phi_1$ can be further expressed as

$$\phi_1 = CA(t)\sin(kx)\exp(-kz) \tag{4}$$

with an unknown constant $C$ to be determined by matching the boundary condition. On the surface of the conductive film itself, the electrostatic potential must have a constant zero due to the grounding (see Fig. 1), thus the perturb $\phi_1$ given in (4) can be determined as

$$\phi_1 = 4\pi\sigma_0 A(t)\sin(kx) \tag{5}$$

for $z = 0$. As a result, the first-order expansion of the surface electrostatic traction (Maxwell stress tensor) leads to

$$\begin{aligned} p_e(w) = -\frac{E^2}{8\pi} &\approx -2\pi\sigma_0^2 - [k\sigma_0\phi_1]_{z=0} \\ &= -2\pi\sigma_0^2 - 4\pi\sigma_0^2 kw, \end{aligned} \tag{6}$$

which is closely related to the perturb amplitude and the perturb wave number in the asymptotic sense.

### 2.2 Stress and velocity field in a layer of incompressible newtonian flow

The motion of the viscous layer under consideration is slow such that the inertia can be ignored, thus its equilibrium equations reduce to

$$\sigma_{ij,j} = 0 \quad (i,j = 1,2,3), \tag{7}$$

where $\sigma_{ij}$ is the Cauchy stress tensor. For an incompressible newtonian flow, its constitutive relations are

$$\sigma_{ij} = \eta(v_{i,j} + v_{j,i}) - p\delta_{ij}, \tag{8}$$

where $\eta$ is the kinematic viscosity, $v_i$ the particle velocity of the viscous layer, $p$ the hydrostatic pressure, and $\delta_{ij}$ is the *Kronecker delta* sign ($\delta_{ij} = 1$ in the cases of $i = j$, and $\delta_{ij} = 0$ for other cases). The incompressibility of the viscous layer leads to

$$v_{i,i} = 0 \tag{9}$$

and

$$p = -\sigma_{ii}/3 \tag{10}$$

is the hydrostatic pressure of the flow. Under such assumptions, the flow is referred to as the Stokes flow or the creeping flow.

There exists a trivial solution to (7)–(10) that corresponds to an infinite viscous layer staying on a flat rigid substrate with constant thickness and traction-free surface. Now let us assume a small perturb to this trivial solution at surface of the viscous layer in the $x - z$ plane. Therefore, the perturbed viscous layer is in a state of inplanar-strain deformation in the $x - z$ plane, i.e., $v_x = v_x(x, z, t), v_z = v_z(x, z, t)$, and $v_y = 0$. Because the viscous layer is infinite, the perturbed field has its translational symmetry along the surface such that the location of the origin of $x$-coordinate is arbitrary. Thus, the tractions acting on the flow surface are assumed to take the forms:

$$\sigma_{zz}(z = H) = q_0(t)\sin(kx), \tag{11}$$

$$\sigma_{zx}(z = H) = \tau_0(t)\cos(kx), \tag{12}$$

where $H$ is the thickness of the viscous layer, $k$ the perturb wave number, $q_0(t)$ and $\tau_0(t)$ are the time-dependent amplitudes of the surface tractions. The velocities of the flow at the bottom of the viscous layer are set to be zero to favor the nonsliding condition. Under above boundary conditions, the explicit solution to (7)–(10) is available in the literature [14], of which the velocities of the particles at the top surface of the viscous layer bear the forms:

$$v_z(z = H) = \bar{v}_z(t)\sin(kx), \tag{13}$$

$$v_x(z = H) = \bar{v}_x(t)\cos(kx). \tag{14}$$

Here the velocity amplitudes, $\bar{v}_z(t)$ and $\bar{v}_x(t)$, are linearly correlated to the amplitudes of the surface tractions such that

$$\bar{v}_z(t) = \frac{1}{2\eta k}[\gamma_{11}q_0(t) + \gamma_{12}\tau_0(t)], \tag{15}$$

$$\bar{v}_x(t) = \frac{1}{2\eta k}[\gamma_{21}q_0(t) + \gamma_{22}\tau_0(t)], \tag{16}$$

where the dimensionless coefficients are:

$$\gamma_{11} = \frac{1}{2}\frac{\sinh(2kH) - 2kH}{(kH)^2 + \cosh^2(kH)}, \tag{17}$$

$$\gamma_{22} = \frac{1}{2}\frac{\sinh(2kH) + 2kH}{(kH)^2 + \cosh^2(kH)} \tag{18}$$

and

$$\gamma_{12} = \gamma_{21} = \frac{(kH)^2}{(kH)^2 + \cosh^2(kH)}. \tag{19}$$

The above solution holds in the limiting case of either infinitely thick layer ($kH\rightarrow \infty$) or very thin film ($H \rightarrow 0$).

### 2.3 Deflection of an elastic film

For analysis of wrinkling in a thin elastic film, nonlinear plate theory is generally required in order to capture the finite deflection. While for the first-order instability analysis of wrinkle initiation, classic linear Kirchhoff plate theory is sufficient for preliminary linear perturbation analysis. In such case, the thin film on surface of the viscous layer is subjected to the inplanar membrane force $N$, the electrostatic surface pressure $p_e$, the net normal fluid pressure $q$, and the shear traction $\tau$ at the film bottom surface. Under the plane-strain condition, the equilibrium equations within the framework of classic linear Kirchhoff plate theory [27] lead to

$$q + p_e = -\frac{Eh^3}{12(1-\nu^2)}\frac{\partial^4 w}{\partial x^4} + N\frac{\partial^2 w}{\partial x^2} + \tau\frac{\partial w}{\partial x}, \tag{20}$$

$$\tau = \frac{\partial N}{\partial x}, \tag{21}$$

where $w$ is the deflection of the thin elastic film, $h$ the film thickness, $E$ the Young's modulus, and $\nu$ is Poisson's ratio. Through Hooke's law, the membrane force $N$ relates to the inplanar displacement $u$ in the $x$-direction. If the thin film is initially compressed with a biaxial strain $\varepsilon_0$ induced by cooling for instance and the inplanar displacement is set to be zero at its initial state, the membrane force $N$ may be expressed as

$$N = \frac{E\varepsilon_0 h}{1-\nu} + \frac{Eh}{1-\nu^2}\frac{\partial u}{\partial x}. \tag{22}$$

Accordingly, assume the displacements of the thin elastic film have a compatible perturb such that

$$w(x, t) = A(t)\sin(kx), \tag{23}$$

$$u(x, t) = B(t)\cos(kx), \tag{24}$$

where $A(t)$ and $B(t)$ are two arbitrary small amplitudes. Substituting (23) and (24) into (22), (21) and (20), and keeping only the leading terms with either $A(t)$ or $B(t)$, one obtains the following relations:

$$N = \frac{E\varepsilon_0 h}{1-\nu} - \frac{E(kh)}{1-\nu^2}B(t)\sin(kx), \tag{25}$$

$$\tau = -\frac{E(kh)k}{1-\nu^2}B(t)\cos(kx), \tag{26}$$

$$q + p_e = -\frac{E(kh)k}{12(1-\nu^2)}[12(1+\nu)\varepsilon_0 + (kh)^2]A(t)\sin(kx). \tag{27}$$

Thus, within the framework of linear perturbation theory, there exist three linear relations that determine the forces $N$, $\tau$, and $q + p_e$ through the film displacement $u$ and $w$, respectively.

### 2.4 Governing equations of a charged elastic film on a layer of incompressible newtonian flow

From velocities (13) and (14) and displacements (23) and (24), two kinematic relations are implied:

$$\bar{v}_z = \frac{\mathrm{d}A(t)}{\mathrm{d}t} \quad \text{and } \bar{v}_x = \frac{\mathrm{d}B(t)}{\mathrm{d}t}. \tag{28}$$

Therefore, by relating (11) and (12) to (26) and (27), one can formulate the following relations of the thin film system as

$$\tau_0(t) = -\frac{E(kh)k}{1-\nu^2}B(t), \tag{29}$$

$$q_0(t) = \left\{-\frac{E(kh)}{12(1-\nu^2)}[12(1+\nu)\varepsilon_0 + (kh)^2] + 4\pi\sigma_0^2\right\} kA(t). \tag{30}$$

Consequently, substituting (28)–(30) into (15) and (16) leads to the final set of equations that governs the perturb growth of the thin film system such that

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} A(t) \\ B(t) \end{bmatrix} = \begin{bmatrix} \alpha & \gamma_{12}/\gamma_{22}\beta \\ \gamma_{12}/\gamma_{11}\alpha & \beta \end{bmatrix}\begin{bmatrix} A(t) \\ B(t) \end{bmatrix}, \tag{31}$$

where

$$\alpha = \left\{-\frac{E(kh)}{24(1-\nu^2)}[12(1+\nu)\varepsilon_0 + (kh)^2] + 2\pi\sigma_0^2\right\}\gamma_{11}/\eta, \tag{32}$$

$$\beta = -\frac{E(kh)\gamma_{22}}{2\eta(1-\nu^2)} \tag{33}$$

and coefficients $\gamma_{11}$, $\gamma_{12}$, and $\gamma_{22}$ are determined by (17)–(19). Similar relations have been derived in the literature in the case of wrinkling of charge-free elastic films [14]. The general solution of (31) can be expressed as

$$\begin{bmatrix} A(t) \\ B(t) \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}\exp(st), \tag{34}$$

where $A$ and $B$ are the two unknown amplitudes, and $s$ is the wrinkle growth rate. Furthermore, substitution of (34) into (31) yields an eigenvalue problem as

$$\begin{vmatrix} \alpha - s & \gamma_{12}/\gamma_{22}\beta \\ \gamma_{12}/\gamma_{11}\alpha & \beta - s \end{vmatrix} = 0, \tag{35}$$

which results in two wrinkle growth rates such that

$$s_1 = \left\{(\alpha+\beta) + \sqrt{(\alpha+\beta)^2 + 4\alpha\beta[\gamma_{12}^2/(\gamma_{11}\gamma_{22}) - 1]}\right\}/2, \tag{36}$$

$$s_2 = \left\{(\alpha+\beta) - \sqrt{(\alpha+\beta)^2 + 4\alpha\beta[\gamma_{12}^2/(\gamma_{11}\gamma_{22}) - 1]}\right\}/2. \tag{37}$$

The signs of $s_1$ and $s_2$ are determined by the wrinkle mode number $k$, initial film inplanar strain $\varepsilon_0$, surface charge density $\sigma_0$, film Young's modulus $E$, fluid viscosity $\eta$, and system geometries $h$ and $H$. Positive sign of $s_1$ and $s_2$ accords to the exponential growth of the perturb (i.e., the initial flat film is unstable); while negative sign corresponds to the exponential decay of the perturb (i.e., the initial flat film is stable).

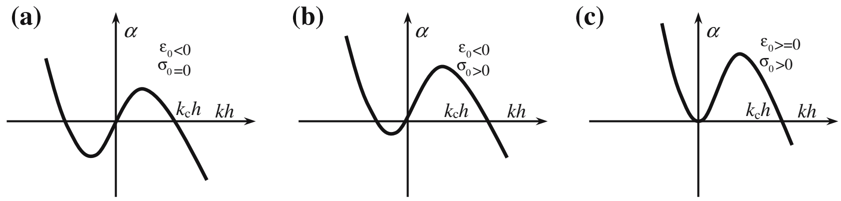


**Fig. 2** Schematic variation of the $\alpha$-value versus the wrinkle mode number $k$. **a** Charge-free elastic films in compression; **b** Charged elastic films in compression; **c** Charged elastic films in tension

## 3 Analysis of wrinkle modes

In relation (33) it always holds $\beta < 0$. Thus, for any given wrinkle mode $kH$ the following relation is always sustained:

$$\gamma_{12}^2/(\gamma_{11}\gamma_{22})-1=\frac{-1}{4\gamma_{11}\gamma_{22}}\frac{\sinh^2(2kH)-(2kH)^2-4(kH)^4}{\cosh^2(2kH)+(2kH)^2}<0. \tag{38}$$

Furthermore, in relations (36) and (37), when $\alpha < 0$, $s_1 < 0$ holds, i.e., perturb relating $s_1$ decays exponentially; while for $\alpha > 0$, $s_1 > 0$ holds, i.e., perturb relating $s_1$ grows exponentially. However, perturb relating $s_2$ always decays since $s_2 < 0$ always holds for any given $\alpha$-value. The critical wrinkle mode number $k_c$ can be determined by letting $\alpha = 0$ in (32) such that

$$-\frac{E(kh)}{24(1-\nu^2)}[12(1+\nu)\varepsilon_0+(kh)^2]+2\pi\sigma_0^2=0, \tag{39}$$

which corresponds to the wrinkle growth rate $s_1 = 0$.

Now, let us first consider two limiting cases of a charge-free film and an initially charged traction-free film, respectively. For a charge-free film (i.e., $\sigma_0 \to 0$), the critical wrinkle mode number $k_c$ is

$$k_c h = \sqrt{-12(1+\nu)\varepsilon_0}, \tag{40}$$

which is independent of the properties of the viscous layer and the geometries of the system [14]. In this case, when $k > k_c$, $s_1 < 0$ and the perturb decays; while $k < k_c$, $s_1 > 0$ and the perturb grows exponentially. Relation (40) indicates that the wrinkle occurs only for compressed film (i.e., $\varepsilon_0 < 0$). Furthermore, for an initially charged traction-free film ($\varepsilon_0 = 0$), the resulting critical wrinkle mode number from (39) is

$$k_c h = 2[6\pi(1-\nu^2)\sigma_0^2/E]^{1/3}. \tag{41}$$

In this case, when $k > k_c$, $s_1 < 0$ and the perturb decays; however, when $k < k_c$, $s_1 > 0$ and the perturb grows exponentially. As a result, relation (41) shows that initially charged traction-free film is always unstable, and the critical wrinkle mode number is closely related to the charge density, film elasticity, as well as film thickness. Therefore, external electric field is capable of modulating the wrinkle mode of thin metallic films.

Moreover, in a general sense, (39) gives one positive root $k_c$ for $\alpha = 0$ in (32). The general root distribution of (39) under varying condition is illustrated in Fig. 2 (all $k$-values with physical meanings should be positive). The normalized wrinkle growth rate $s_1\eta/E$ versus the wrinkle mode number $kh$ for several thickness ratios $H/h$ are plotted in Fig. 3. In the numerical procedure, Poisson's ratio of the thin metallic film is selected as 0.3, and the magnitudes of the compressive and tensile strains are both selected as 0.012. For comparison, Fig. 3a shows the results for a compressed charge-free film given in the literature [14]. Figure 3b, c shows the results for a charged film in compression. It shows that surface charges contribute the wrinkle evolution. Figure 3d presents the results of a charged film in tension. It indicates that surface charges may even lead to the wrinkle of a stretched film. As a matter of fact, the wrinkle mode and its growth rate are governed through the coupling effect of the surface charges and the initial film strains.

Now let us further consider two limiting cases of the viscous layers at thick and thin thicknesses, respectively. For a very thick viscous layer (i.e.,

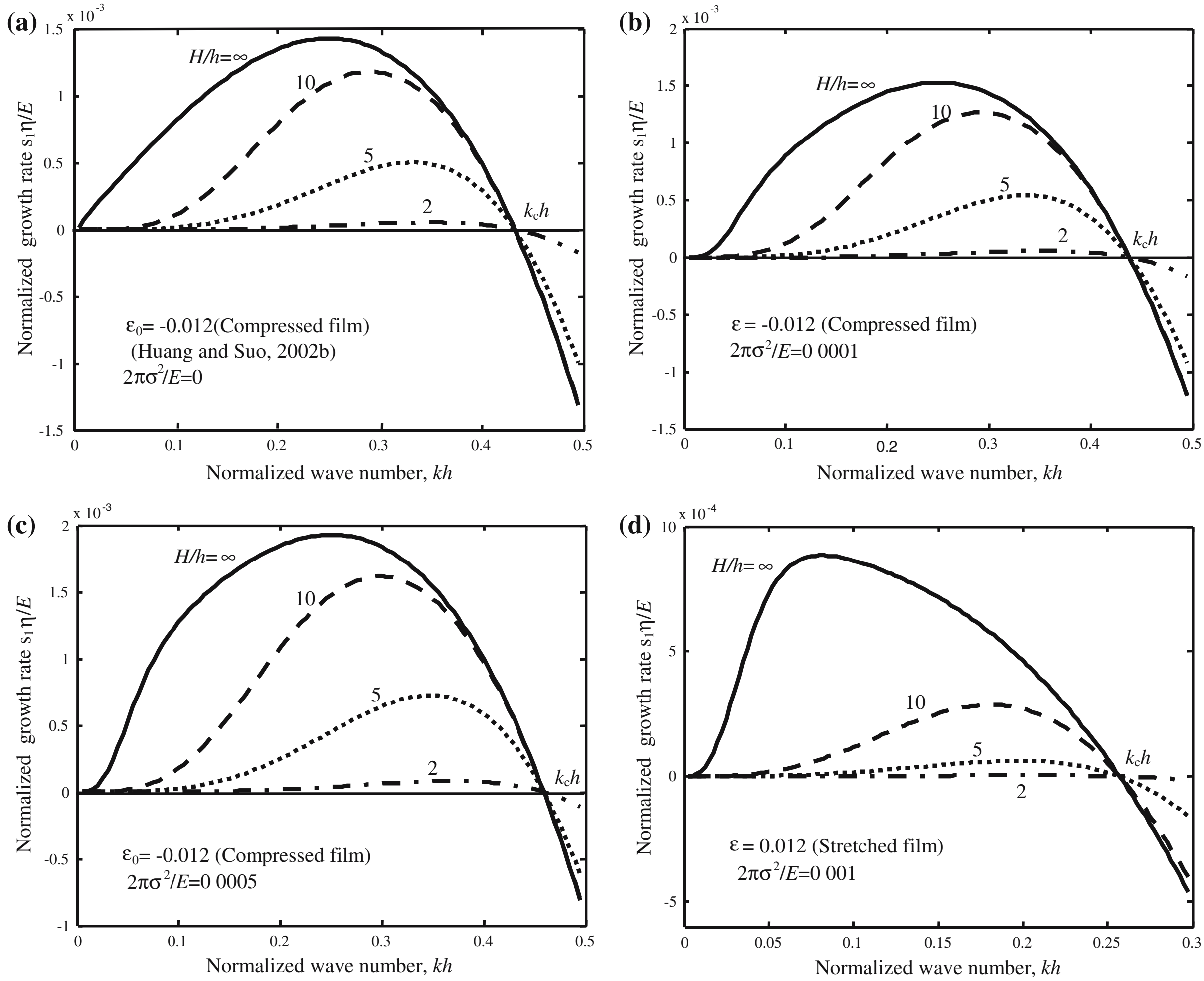


**Fig. 3** The wrinkle growth rate $s_1$ versus the mode number $k$ with varying thickness ratio. **a** Charge-free elastic films in compression; **b** and **c** Charged elastic films in compression; **d** Charged elastic films in tension

$H/h \to \infty$), the limiting wrinkle growth rate $s_1$ can be extracted from relation (36) such that

$$s_1\eta/E = \left\{-\frac{kh}{24(1-\nu^2)}[12(1+\nu)\varepsilon_0 + (kh)^2] + 2\pi\sigma_0^2/E\right\}. \quad (42)$$

For a very thin viscous layer (i.e., $H/h \to 0$), the corresponding wrinkle growth rate $s_1$ is

$$s_1\eta/E = \frac{2(kH)^3}{3}\left\{-\frac{kh}{24(1-\nu^2)}[12(1+\nu)\varepsilon_0 + (kh)^2] + 2\pi\sigma_0^2/E\right\}. \quad (43)$$

Furthermore, from Fig. 3, it can be observed that the wrinkle growth rate $s_1$ tends to zero when the mode number $k$ tends to both zero and $k_c$. Therefore, a stationary point must exist in the range (0, $k_c$). At this point, $s_1$ reaches its peak value, i.e., the fastest wrinkle growth rate $s_m$. This value may be determined by directly letting $\partial s_1/\partial k = 0$ in (36), and the resulting $k_m$ is the wrinkle mode number with fastest growth rate. This wrinkle mode is the one to be expected in experiment. Figure. 4 shows $k_m h$ and corresponding $s_m\eta/E$ as functions of the compressive strain $\varepsilon_0$ for several film thickness ratios. Figure 4a, b are the results for charge-free films, and Fig. 4c, d are those for charged films. From Fig. 4a, d, it can be found that $k_m h$ grows with

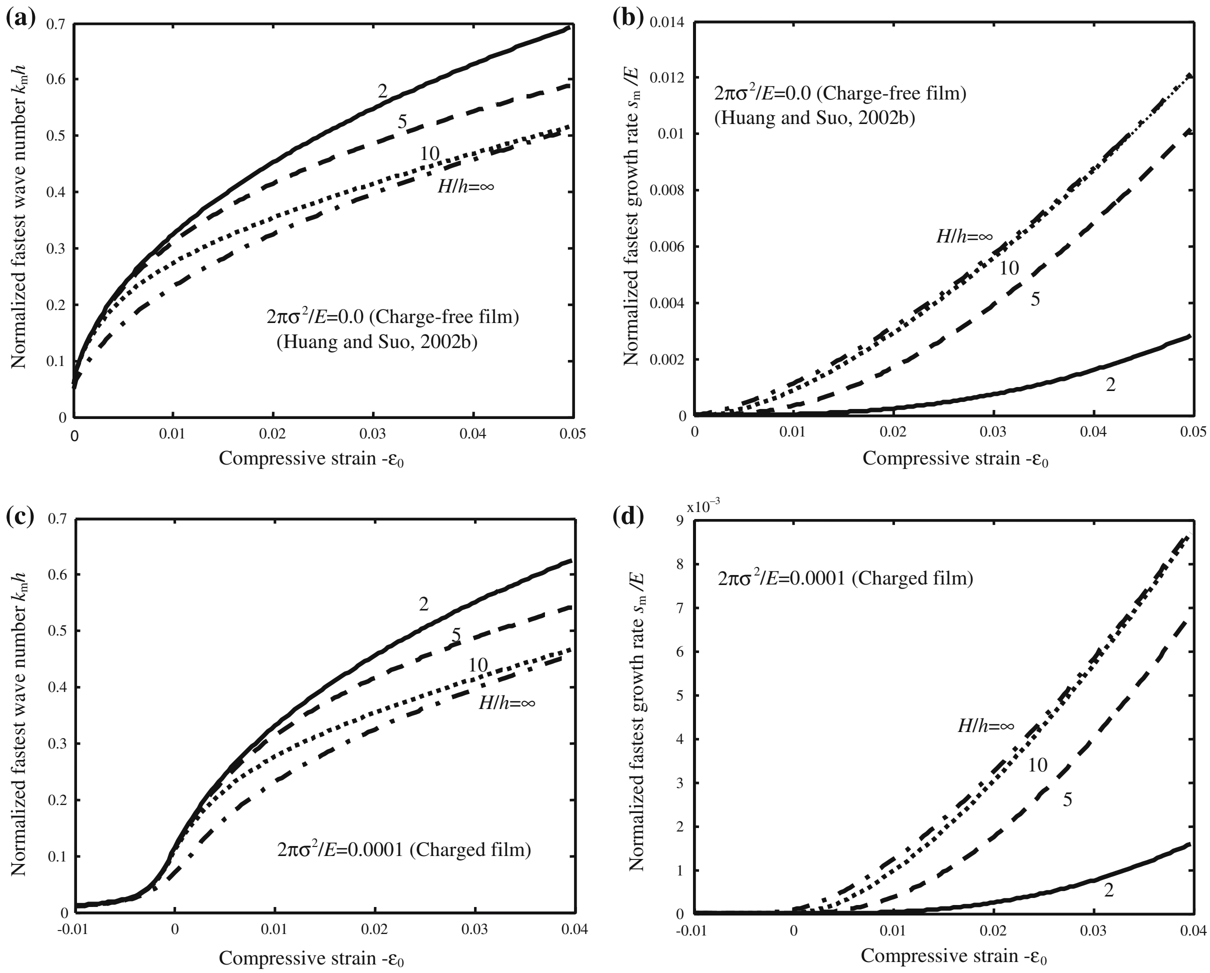


**Fig. 4** The fastest growth mode number $k_m$ and corresponding wrinkle growth rate $s_m$ versus the compressive strain with varying thickness ratio. **a** and **b** Charge-free elastic films in compression; **c** and **d** Charged elastic films in compression and in tension

the increase of $\varepsilon_0$; while it decreases with the increase of the thickness ratio $H/h$. The corresponding $s_m\eta/E$ increases with the increase of both $\varepsilon_0$ and $H/h$. Surprisingly, Fig. 4c, d also indicate that surface charges may result in the wrinkling of a stretched film.

In the limiting case of very thick viscous layer ($H/h \to \infty$), $s_m$ and corresponding $k_m$ are determined from (42) as

$$k_m h = 2\sqrt{-\varepsilon_0(1+\nu)},$$

$$s_m\eta/E = \frac{2\sqrt{-\varepsilon_0(1+\nu)}}{3(1-\nu)} + \frac{2\pi\sigma_0^2}{E}. \tag{44}$$

For very thin fluid layer ($H/h \to 0$), $k_m$ relating the fastest wrinkle growth rate $s_m$ is determined through solving the characteristic equation:

$$(kh)^3 + 8(1+\nu)\varepsilon_0(kh) + 8\pi(1-\nu^2)\sigma_0^2 = 0. \tag{45}$$

In the limiting case of charge-free film ($\sigma_0 = 0$), relation (45) reduces to

$$k_m h = 2\sqrt{-2\varepsilon_0(1+\nu)} \tag{46}$$

and the resulting growth rate is

$$s_m\eta/E = 16[-(1+\nu)H/h]^3/[9(1-\nu^2)]. \tag{47}$$

## 4 Concluding remarks

Wrinkling of charged thin metallic films on viscous layers has been studied within the framework of linear perturbation theory. The critical wrinkle mode number has been determined as the bifurcation point of the elastic instability of the thin films. The fastest wrinkle growth rate and the corresponding mode number have been determined as functions of the film surface charge, film elasticity and thickness, mismatch strain, as well as viscosity and thickness of the viscous layer. For charged thin metallic films, the elastic wrinkle may happen not only for compressed films but also for stretched films. Therefore, external electric field can be utilized as one of the feasible manners to modulate the wrinkle mode of thin metallic films on viscous layers such as compliant polymeric layers used extensively in surface coatings, lithography, and semiconductor engineering, etc. General analysis and simulation of elastic instability and wrinkle pattern evolution of charged thin films in the 2D case can be developed using the present scheme and will be reported elsewhere.

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