

Talent Search

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

https://www.ndsu.edu/math/ongoing_events/nd_talent_search/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by January 31, 2018, using the form on the website. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the second set of problems:

1. An aquarium contains 200 fish. 99% of them are goldfish. We want to transfer some goldfish to a second aquarium, until exactly 98% of the fish in the first aquarium are goldfish. How many goldfish do we need to take out?
2. Two players have one stack with 2017 counters. They take turns in taking from the stack any number of counters they want between 7 and 17 (including both 7 and 17). If at some point in the game, fewer than 7 counters remain, the player whose turn it is to play can take all remaining counters. The player to empty the stack wins. What is the winning strategy? Is it best to be first or second player?

3. Let m, n be natural numbers such that $\frac{m^2}{n^2} < 2$.

(a) Show that $\frac{(m + 2n)^2}{(m + n)^2} > 2$.

(b) Show that $\frac{(m + 2n)^2}{(m + n)^2} - 2 < 2 - \frac{m^2}{n^2}$.

(c) Start with $\frac{7}{5}$ and use (a) and (b) to find a sequence of fractions that approximate $\sqrt{2}$. Stop when your approximation agrees with $\sqrt{2}$ up to the fifth decimal digit.

4. We send a secret message as follows:

(a) First, we associate a number to each letter of the alphabet, with $A = 0, B = 1, \dots, Z = 25$. In fact, for numbers are larger than 25, we rewrap them and consider that 26 is again A , 27 is again B , etc.

(b) Divide your message in pairs of letters, transform them into pairs of numbers (x, y) .

Example: MEET ME AT FOUR, gets divided into ME ET ME AT FO UR, which gives the pairs $(12, 4), (4, 19), (12, 4), (0, 19), (5, 14), (20, 17)$.

(c) Replace each pair (x, y) by the new pair $(3x + 6y, x + 3y)$. Turn this back to letters, make it into a single “word”, and send.

Your friend uses this system to send you the message MIVSPTLZ. Can you decode it?

5. Two mirrors form a 30 degree angle. A light beam enters this angle parallel to one of the sides, and is reflected several times (remember that the angle of incidence is equal to the angle of reflection). How many times will it reflect before leaving the angle? What if the angle between the mirrors is 20 degrees?