The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at https://www.ndsu.edu/math/ongoing_events/nd_talent_search/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; finding a good solution to a problem is always an achievement. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by March 31, 2019, using the form on the website. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the third set of problems:

1. Show that if \(a, b, c\) are such numbers that \(a + b + c = 0\) then \(ab + ac + bc \leq 0\).

2. How many 6 character passwords can be made using only A, B, C, D, E, 1, 2, 3, 4, 5 in each one of the following situations?

   (a) The password contains 4 letters and 2 digits (in any order and repeats allowed).
   (b) The password must contain at least one letter and at least one digit (repeats allowed).
   (c) No character is used more than once.
   (d) No two letters are adjacent, no two digits are adjacent, and no character is used more than once.
3. Consider a trapezoid with base $AD$ and parallel top $BC$. The length of $AB$ is twice the length of $BC$. Let $P$ be the midpoint of the side $AB$, and let $R$ be a point on the side $CD$ such that $2 \text{length}(CD)=3 \text{length}(RD)$. Let $Q$ be the point of intersection of the lines $PD$ and $AR$. What is the area of the triangle $AQP$?

4. We have two balls of equal mass. Ball $A$ is at rest at distance 1 from a wall. Ball $B$ is approaching $A$, perpendicularly to the wall, at constant speed equal to 1. When ball $B$ hits ball $A$, $B$ stops and $A$ starts moving toward the ball with speed 1 (the collision is elastic, so no energy is lost). $A$ hits the wall, bounces back at speed 1, and collides with $B$. At that moment, $A$ stops, and $B$ starts moving away from the wall at speed 1. In this process, there have been a total of 3 collisions.

Now suppose that the mass of $B$ is 10 times bigger than the mass of $A$, and $A$ is at distance 100 from the wall. When $B$ hits $A$, it transfers part of its speed to $A$, which starts moving towards the wall, but $B$ itself continues moving in the direction of the wall at a smaller speed. $A$ reaches the wall first, bounces back, and hits $B$. A new transfer of energy is produced, $A$ moves again towards the wall, and so does $B$, with an even lower speed. The process continues till eventually $B$ changes direction and moves away from $A$ and the wall. What is the total number of collisions in this setting?

Hints: You will need to use the laws of conservation of kinetic energy and momentum. You don’t need to compute the distances to the wall at each collision.