DEPARTMENT OF MATHEMATICS ASSESSMENT REPORT,
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CONTENTS

Part I. Department of Mathematics Assessment Strategy 3
  I.1. Introduction 3
  I.2. Assessment Scheme 3
  I.3. Rationale 4
  I.4. Changes to the Assessment Strategy 5
  I.5. Structure of the report 5
  I.6. Future assessment activities 5

Part II. Overview Statement 6

Part III. Data from Final Examinations, Analysis and Reflections 7
  III.1. Math 104, Finite Mathematics, first section 7
  III.2. Math 104, Finite Mathematics, second section 12
  III.3. Math 104, Finite Mathematics, third section 15
  III.5. Math 128, Introduction to Linear Algebra, first section 24
  III.6. Math 128, Introduction to Linear Algebra, second section 27
  III.7. Math 128, Introduction to Linear Algebra, third section 31
  III.8. Math 128, Introduction to Linear Algebra, fourth section 33
  III.9. Math 129, Basic Linear Algebra, first section 35
  III.10. Math 129, Basic Linear Algebra, second section 38

Date: February 14, 2011.
| III.11. | Math 129, Basic Linear Algebra, third section | 42 |
| III.12. | Math 129, Basic Linear Algebra, fourth section | 46 |
| III.13. | Math 129, Basic Linear Algebra, fifth section | 49 |
| III.14. | Math 129, Basic Linear Algebra, sixth section | 52 |
| III.15. | Math 129, Basic Linear Algebra, seventh section | 56 |
| III.16. | Math 129, Basic Linear Algebra, eighth section | 60 |
| III.17. | Math 146, Applied Calculus I, first section | 63 |
| III.18. | Math 146, Applied Calculus I, second section | 66 |
| III.19. | Math 147, Applied Calculus II, first section | 70 |
| III.20. | Math 147, Applied Calculus II, second section | 73 |
| III.21. | Math 165, University Calculus I, first section | 76 |
| III.22. | Math 165, University Calculus I, second section | 79 |
| III.23. | Math 165, University Calculus I, third section | 83 |
| III.24. | Math 165, University Calculus I, fourth section | 88 |
| III.25. | Math 165, University Calculus I, fifth section | 91 |

**Part IV. Client Department/College Reflection**

**Part V. Mathematics Department Reflection**

**Part VI. Summary**
Part I. Department of Mathematics Assessment Strategy

I.1. Introduction

Departmental assessment consists of several interrelated goals. Departments are encouraged to identify specific course and program objectives and to assess the degree to which students are meeting these objectives. To achieve these goals, the Mathematics Department is continuing the assessment strategy initiated with the January 2009 report. This strategy involves (1) the collection and analysis of summative assessment data from final examinations in certain mathematics courses (rotating annually through a cyclic protocol) and (2) faculty reports of formative assessment activities in these courses.

I.2. Assessment Scheme

We begin with the following model for course objectives in mathematics. We consider the list of topics from the NDSU Course Bulletin for a given course. The desired outcomes of this course then take the following form: Students should be able to demonstrate through written assignments, quizzes and exams that they have developed an understanding of the course topics. For instance, a student’s success in Math 165 will be measured by his or her ability to solve problems related to the topics of limits, continuity, differentiation, Mean Value Theorem, integration, Fundamental Theorem of Calculus and applications. Instructors are encouraged to augment this list as necessary.

To assess students’ degrees of success in mathematics courses, we collect and analyze numerical data from written final examinations given in all mathematics courses. Our procedures for collection and analysis of these data are described as follows.

For each course being assessed for this report, the instructor supplied the committee with the following:

(a) A copy of the final examination;
(b) A list of the course objectives being assessed by each final exam question;
(c) For each student, the numerical scores for the individual questions on the exam; and
(d) A description of formative assessment activities conducted for the course.

In addition, instructors for the course sections being assessed were asked to consider the following rubric for assessing the degree of success in solving a given problem:

A. Completely correct;
B. Essentially correct—student shows full understanding of solution and only makes a minor mistake (e.g., wrong sign when calculating a derivative or arithmetic error);
C. Flawed response, but quite close to a correct solution (appears they could do this type of problem with a little review or help);
D. Took some appropriate action, but far short of a solution;
E. Blank, or nothing relevant to the problem.

Using a scale of 0 to 100, these instructors were asked to describe the range of scores which (according to their personal rubric) falls into each of the above categories. For example, one possible response is the following:

A. 100  
B. 80–99  
C. 50–79  
D. 30–49  
E. 0–29.

For each final exam question, the committee calculated the percentages of students in the class whose numerical score satisfied the instructor’s degree-of-success criteria. The committee then provided the instructor with these data and asked for the instructor’s reactions to the data. Specifically, the committee asked each instructor to answer the following two questions:

1. What did you learn from these data?  
2. What will you do differently as a result of what you learned?

Next, the draft report (including the data and the instructors’ responses) was presented to our client departments/colleges. We asked for their responses to the data via the following prompts:

1. Given the data included in the report, please indicate how well you feel that these courses are serving your students.  
2. Given the data included in the report, please list any changes you would like made in these courses.  
3. Given the data included in the report, please list any questions or concerns that you have about these courses.

Finally, the draft report (including the data, the instructors’ responses, and the responses of the client departments/colleges) was presented to the mathematics department with a request for reactions.

I.3. Rationale

We choose to focus primarily on scores from final examinations for several reasons. First, because final exams in mathematics are often cumulative in nature, data from these exams should contain information about each of the desired learning outcomes associated with a given course. Second, assessment requires a certain amount of data. However, we need to restrict ourselves to a reasonable amount that does not cause an undue burden for the department or for the Assessment Committee.
We have included reports of formative assessment activities at the request of the University Assessment Committee (UAC) and because these activities are important measures of the mathematics faculty’s commitment to student learning. It should be noted that most faculty have only described the formative assessment activities they conducted; they did not report the outcomes of this assessment. We feel that this fits with the intent of formative assessment.

I.4. Changes to the Assessment Strategy

In light of the UAC’s review of the Mathematics Department’s most recent assessment report, we have made several changes in our assessment strategy.

(1) We are assessing all mathematics courses that satisfy general education requirements, i.e., Math 104, 146, and 165; and
(2) We are soliciting responses to the data for these courses from client departments.

Note that the UAC did not explicitly request item (2). Given the number of service courses taught by the Mathematics Department, though, this is a natural course of action.

I.5. Structure of the report

Part I of this report is a description of the Mathematics Department’s assessment strategy. Part II contains an overview of the report. Part III contains the data (both quantitative and qualitative) collected by the committee. It is organized as follows. Each section of this part of the report is devoted to a specific section of one of the courses assessed. Within each section, the first subsection contains the final examination questions; after each question, we have listed the outcomes assessed by that question (as indicated by the instructor) and the degree-of-success data (as calculated by the committee). The second subsection contains the instructor’s reaction to the data. The final subsection contains a description of the instructor’s formative assessment activities for the course. (For courses where no formative assessment was conducted, this subsection is omitted.)

Part IV of the report contains reactions of client departments/colleges to the report.
Part V of the report contains reactions of individual faculty members to the report.
Part VI is a concluding summary.

I.6. Future assessment activities

The Assessment Committee plans to continue collecting and analyzing data in this fashion for the next several years. This strategy will allow the department to amass a nontrivial
amount of data; we expect that this will enable us to identify trends within the department. The committee plans to assess certain courses each year according to a cyclic protocol. Specifically,

- The January 2009 report assessed Math 165–266.
- The current report assesses Math 104, 128, 129, 146, 147, and 165.
- The January 2012 report will assess Math 101–107, 146, and 165.
- The January 2013 report will assess mathematics graduate courses and Math 104, 146, and 165.

Then the cycle will repeat.

In future reports, we plan to consider further activities including the following:

(i) Establish better protocols for efficient but thorough assessment of multiple sections of courses that use a common final exam;
(ii) Establish explicit desired outcomes for students majoring or minoring in mathematics, and identify methods for assessing these outcomes;
(iii) Establish explicit desired outcomes for students in mathematics courses who are not majoring or minoring in mathematics, and identify methods for assessing these outcomes; and
(iv) Establish explicit desired outcomes for students completing graduate degrees in mathematics, and identify methods for assessing these outcomes.

**Part II. Overview Statement**

The Department of Mathematics Assessment Committee collected numerical scores for individual questions from final examinations from Fall 2008 and Spring 2009 for all sections of Mathematics courses numbered 104, 128, 129, 146, 147, and 165. These data were analyzed according to a rubric determined in collaboration with the individual instructors. The instructors also described formative assessment activities for the courses being assessed. The instructors and other faculty members were asked to reflect on the data.
Part III. Data from Final Examinations, Analysis and Reflections

III.1. Math 104, Finite Mathematics, first section

III.1.1. Final exam.

Question 1.

Pets R Us is a manufacturer of flea collars. The weekly cost and revenue from producing and selling a certain kind of collar are given by \( C(x) = 35 + 2.5x \) and \( R(x) = x(12 - 0.42x) \), respectively, where \( x \) is in hundreds of collars and \( C(x) \) and \( R(x) \) are in hundreds of dollars. Assume that \( 0 \leq x \leq 20 \).

a.) Determine the level(s) of production, to the nearest collar, at which the company breaks even.

b.) Determine the level of production, to the nearest collar, which generate a profit of $1500.

Outcomes assessed: Cost/Profit/Revenue

Degree of success: A 24%  B 11%  C 15%  D 37%  E 13%

Question 2.

A class of 8 students took an exam and the scores were as follows: 83, 79, 49, 99, 80, 38, 100, 72.

a.) What was the mean score on the exam?

b.) Find the variance and standard deviation for the exam scores. If necessary to round, give your answer to three decimal places.

Outcomes assessed: Descriptive statistics

Degree of success: A 23%  B 9%  C 21%  D 47%  E 0%

Question 3.

Three years ago, Jane deposited $4700 into an account earning 3.7% compounded quarterly. She has not made any deposits or withdrawals since she opened the account.

a.) How much money is in the account now?

b.) How long will it take until she has at least $10,000 in the account? (Be sure to include units and show how you arrived at your answer.)

Outcomes assessed: Mathematics of finance.

Degree of success: A 18%  B 26%  C 13%  D 43%  E 0%
Question 4.
Given the following system of linear equations:

\[
\begin{align*}
3x + 2y - z &= 12 \\
-7x + y &= 27 \\
9x - 2y + z &= 36
\end{align*}
\]

a.) Write the augmented matrix for this system. List a sequence of row operations to obtain a 1 in row 1, column 1 and two 0s in the rest of the first column.
b.) Is (4, 1, 2) a solution to the system? Justify your answer.

Outcomes assessed: Systems of linear equations
Degree of success: A 41%  B 17%  C 24%  D 13%  E 5%

Question 5.
If possible, maximize and minimize \( C = 17x + 18y \), subject to the constraints (inequalities) given below. Be sure to show a neat sketch of the feasible region, labeling all corner points, and show how you arrive at your conclusion.

\[
\begin{align*}
x + 2y &\leq 150 \\
-3x + 4y &\leq 100 \\
x, y &\geq 0
\end{align*}
\]

Outcomes assessed: Systems of linear inequalities, linear programming
Degree of success: A 40%  B 8%  C 31%  D 13%  E 8%

Question 6.
a.) At a large dinner party, there are 3 kinds of appetizers, 5 different entrées, and 4 kinds of dessert. How many different meals are possible if a person chooses one item from each category (appetizer, entrée, dessert).
b.) Jeff has three books by each of his favorite four authors. In how many ways can the books be placed on a shelf if books by the same author must be together?

Outcomes assessed: Elementary probability
Degree of success: A 11%  B 3%  C 42%  D 31%  E 13%

Question 7.
Luke has the top drawer of his desk filled with ink pens. There are 12 black ones, 9 blue ones, and 17 red ones. He reaches in and grabs six pens at random from the drawer.

a.) What is the probability that all six are blue?
b.) What is the probability that at least one of the six is red?
Outcomes assessed: Elementary probability
Degree of success: A 11%  B  3%  C 17%  D 22%  E 47%

Question 8.
From a jar containing 18 black jelly beans, 10 red jelly beans, and 12 yellow jelly beans, two jelly beans are taken out at random and eaten, one at a time. Draw a probability tree and use it to answer the following questions.

a.) What is the probability that a red one was eaten first, followed by a yellow one?

b.) What is the probability that the second one was black?

Outcomes assessed: Elementary probability
Degree of success: A 25%  B  4%  C 7%  D 35%  E 29%

Question 9.
Suppose that Neil borrows $125,000 for a new house. He signs a 25-year mortgage at 5.5% compounded monthly.

a.) What is Neil’s monthly payment, to the nearest cent?

b.) How much total interest will Neil pay?

c.) What is the unpaid balance of the loan after 12 years?

Outcomes assessed: Mathematics of finance
Degree of success: A 19%  B  3%  C 31%  D 36%  E 11%

Question 10.
Suppose that the number of fleas on my Great Dane, Sparky, is increasing linearly with respect to time. There were 87 fleas on Sparky at noon yesterday, and four hours later there were 143 fleas on him.

a.) Write a formula which gives the number of fleas, $y$, on Sparky, $x$ hours after noon yesterday.

b.) If the number of fleas continued to increase at the same rate, then how many fleas were on Sparky at 11:30 last night?

Outcomes assessed: Systems of linear equations
Degree of success: A 55%  B  4%  C 19%  D 9%  E 13%

Question 11. (Bonus)
Jessica and Jon are at a dinner party. There are 12 people (including Jessica and Jon) at the party sitting around a large round table. People were assigned a seat at random. What is the probability that Jessica and Jon are sitting next to each other?
Outcomes assessed: Elementary probability

Degree of success: A 8%  B 0%  C 3%  D 31%  E 58%

III.1.2. Reflection.

1. What did you learn from these data?

Instructor 1. I thought the results were interesting as far as the breakdown of how my students did on each problem on the final exam. But they weren’t very surprising. There was really only one question that students didn’t do as well as I would’ve predicted. And the success rate was the worst on the material that I consider to be the most difficult.

Instructor 2. This data was able to show us, the math department, that the students in Math 104 take a final exam each semester that results in mixed messages on the success of each topic taught. It may be that the final exam week creates extra stress and/or pressure on the student or that they are not mastering the topics as well as we may like them to. The specific topics I see room for improvement are the Cost/Profit/Revenue graphs and questions about the graphs and Higher Level probability questions such as multiple events and the use of permutations and combinations. They seem to do well with basic linear graphs and systems of equations but also struggle with setting up linear programming problems. I learned from this data that there is always room for improvement on the part of the instructors methods as well as the students time and effort on homework and study skills - set forth each week throughout the semester. Consistent practice and a sense of openness are key to the success of the student. Comfortable communication practices (not afraid to ask a question) and a regular homework collection practice is essential to keeping good work ethic among the classroom. This, in turn, should lead to productivity and learning in each class.

2. What will you do differently as a result of what you learned?

Instructor 1. I don’t think I’ll do much differently based on the results.

Instructor 2. As an instructor, I will collect homework more often (every class period - versus once a week). I will encourage the use of a study group on a regular basis (outside of class). Finally, I will share my findings and ideas with my peers in the math department to encourage improvement within all of us. In conclusion, I feel that consistent homework collection, study groups, along with the present activities of an open communication style, an enjoyable classroom experience and the mutual sharing among peers in the math department are the key ingredients to success in learning math at NDSU.

III.1.3. Formative Assessment. Instructor 1. For Math 104, the way I normally go about presenting information is I begin by talking about the topic in question, and do some examples, having the students lead me through the process. After the material has been covered, and the students have had an opportunity to ask questions, I have them
work together in groups on problems to hand in. I grade the group work and hand it back the next time, giving them another chance to ask questions about anything they might have done wrong. Then I give an individual quiz on the material. They generally have plenty of time to talk to each other and to me in order to grasp the material.
III.2. Math 104, Finite Mathematics, second section

III.2.1. Final exam.

Question 1.

Nag’s Head Kite Company has found that their annual cost and revenue from producing and selling a certain type of kite are given by \(C(x) = 58 + 11x\) and \(R(x) = x(52 - 3x)\), respectively, where \(x\) is in thousands of kites and \(C(x)\) and \(R(x)\) are in thousands of dollars. Assume that \(0 \leq x \leq 16\).

a.) Determine the level(s) of production, to the nearest kite, at which the company breaks even.

b.) Determine the level of production, to the nearest kite, which maximizes revenue.

c.) Write the profit function, \(P(x)\), which gives the profit, in thousands of dollars, from selling \(x\) thousands of kites.

Outcomes assessed: Cost/Profit/Revenue

Degree of success: A 5% B 10% C 25% D 20% E 40%

Question 2.

Lyle deposits $4,500 into an account earning 4.2% compounded monthly.

a.) How much interest will Lyle earn if he keeps the money in the account for 6 years?

b.) How long will it take until Lyle has at least $8,000 in the account? (Be sure to include units and show how you arrived at your answer.)

Outcomes assessed: Mathematics of finance

Degree of success: A 15% B 5% C 35% D 15% E 30%

Question 3.

Given the following system of linear equations:

\[
\begin{align*}
-2x + 4y - 6z &= 10 \\
3x + 5y + 12z &= 7 \\
-5x - y + 10z &= 0
\end{align*}
\]

a.) Write the augmented matrix for this system. List a sequence of row operations to obtain a 1 in row 1, column 1 and two 0s in the rest of the first column.

b.) Perform the operations listed in part a.) and write the resulting matrix.

Outcomes assessed: Systems of linear equations, matrices

Degree of success: A 45% B 10% C 10% D 5% E 30%
Question 4.
Sketch the feasible (solution) region for the following system of inequalities. Show scale on your axes, and list all corner points.

\[
\begin{align*}
x + 2y & \leq 150 \\
-3x + 4y & \leq 100 \\
x, y & \geq 0
\end{align*}
\]

Outcomes assessed: Systems of linear inequalities
Degree of success: A 10%  B 25%  C 30%  D 25%  E 10%

Question 5.
Consider the following linear programming problem.
A company manufactures electronic hockey and soccer games, each requiring assembly and testing. Each hockey game requires 2 labor-hours of assembly and 2 labor-hours of testing. Each soccer game requires 3 labor-hours of assembly and 1 labor-hour of testing. The company makes a profit of $9 on each hockey game and $7 on each soccer game. Each day there are 42 labor-hours available for assembly and 26 labor-hours available for testing. How many of each game should they produce each day to maximize profit?

a.) Write the objective function for this problem. Be sure to declare what your variables represent.

b.) Write all of the constraints.

Outcomes assessed: Systems of linear inequalities, linear programming
Degree of success: A 25%  B 0%  C 20%  D 30%  E 25%

Question 6.
Suppose that a hand of 6 cards is dealt at random from a standard deck.

a.) How many possible hands contain exactly 3 kings?

b.) What is the probability that a 6-card hand contains at least one red card?

Outcomes assessed: Elementary probability
Degree of success: A 0%  B 0%  C 5%  D 10%  E 85%

Question 7.
Suppose that there are 15 students in this class and that 4 of you are going to be chosen to be on a committee.

a.) In how many ways can this be done?

b.) What is the probability that YOU will be on this committee?
Outcomes assessed: Elementary probability
Degree of success: A 30%    B 5%    C 5%    D 35%    E 25%

**Question 8.**
Suppose that Jack takes out a 25-year conventional loan of $220,000 for a new house. The interest rate is 5.9%, compounded monthly, and his monthly payments are $1404.05.

a.) How much will Jack pay on the loan?
b.) What is the unpaid balance of the loan after Jack makes his 200th payment?

Outcomes assessed: Mathematics of finance
Degree of success: A 10%    B 0%    C 5%    D 30%    E 55%

**Question 9.**
Suppose that Joe has an investment portfolio that was worth $10,800 in the year 2000. Four years later, the portfolio was worth $16,000. The value of the portfolio increased linearly. Let $V(t)$ represent the value of his portfolio $t$ years after 2000.

a.) Write an equation for $V(t)$.
b.) After how long will the portfolio be worth $23,000? Give your answer to three decimal places.

Outcomes assessed: Combinations and permutations
Degree of success: A 45%    B 15%    C 5%    D 5%    E 30%

**Question 10. (Bonus)**
Jessica and Jon are at a dinner party. There are 10 people (including Jessica and Jon) at the party sitting around a large round table. People were assigned a seat at random. What is the probability that Jessica and Jon are sitting next to each other?

Outcomes assessed: Elementary probability
Degree of success: A 10%    B 0%    C 0%    D 5%    E 85%

**III.2.2. Reflection.**
1. What did you learn from these data?
I thought the results were interesting as far as the breakdown of how my students did on each problem on the final exam. But they weren’t very surprising. There was really only one question that students didn’t do as well as I would’ve predicted. And the success rate was the worst on the material that I consider to be the most difficult.

2. What will you do differently as a result of what you learned?
I don’t think I’ll do much differently based on the results.
III.3. Math 104, Finite Mathematics, third section

III.3.1. Final exam.

Question 1.
Pets R Us is a manufacturer of pet travel kennels. The cost and revenue from producing and selling a certain kind of kennel are given by $C(x) = 250 + 17x$ and $R(x) = x(152 - 11x)$, respectively, where $x$ is in thousands of kennels and $C(x)$ and $R(x)$ are in thousands of dollars. Assume that $0 \leq x \leq 12$.

a.) Determine the level(s) of production, to the nearest kennel, at which the company breaks even.

b.) Determine the maximum possible profit, to the nearest dollar.

Outcomes assessed: Cost/Profit/Revenue
Degree of success: A 19%  B 19%  C 29%  D 22%  E 11%

Question 2.
A group of 9 people took an IQ test and the results were as follows: 132, 112, 98, 124, 95, 125, 106, 100, and 98.

a.) What was the mean score on the exam?

b.) Find the variance and standard deviation for the IQs. If necessary to round, give your answer to three decimal places.

Outcomes assessed: Descriptive statistics
Degree of success: A 33%  B 5%  C 32%  D 28%  E 2%

Question 3.
Morgan deposits $75 each month into an account earning 3.4% compounded monthly.

a.) How much money is in the account after 3 years?

b.) How long will it take until Morgan has at least $8,000 in the account? Be sure to include units and to show how you arrived at your answer.

Outcomes assessed: Mathematics of finance
Degree of success: A 37%  B 9%  C 34%  D 15%  E 5%
Question 4.
Given the following system of linear equations:

\begin{align*}
4x + 2y - 2z &= 10 \\
-7x + z &= -19 \\
8x - 2y + z &= 18
\end{align*}

a.) Write the augmented matrix for this system. List a sequence of row operations to obtain a 1 in row 1, column 1 and two 0s in the rest of the first column. You don’t need to perform the operations.

b.) Is \((3, 1, 2)\) (i.e., \(x = 3, y = 1, z = 2\)) a solution to the system? Justify your answer.

Outcomes assessed: Matrices, systems of linear equations

Degree of success: A 55%  B 8%  C 24%  D 11%  E 2%

Question 5.
If possible, maximize and minimize \(C(x, y) = 17x + 8y\), subject to the constraints (inequalities) given below. Be sure to show a neat sketch of the feasible region, labeling all corner points, and show how you arrive at your conclusion.

\begin{align*}
x + 2y &\leq 150 \\
-3x + 4y &\leq 100 \\
x, y &\geq 0
\end{align*}

Outcomes assessed: Systems of linear equations

Degree of success: A 37%  B 7%  C 31%  D 17%  E 8%

Question 6.

a.) A particular new car model is available with 5 choices of color, 3 choices of transmission, 4 types of interior, and 2 types of engine. How many different variations of this model car are possible?

b.) Jeff has three books by each of his favorite four authors. In how many ways can the books be placed on a shelf if books by the same author must be together.

Outcomes assessed: Permutations and combinations

Degree of success: A 4%  B 7%  C 45%  D 39%  E 5%

Question 7.
From a group of 12 Democrats and 20 Republicans, a 5-person committee is to be chosen at random.

a.) What is the probability that the committee has exactly two Democrats?
b.) What is the probability that at least one committee member is a Republican?

Outcomes assessed: Elementary probability

Degree of success: A 10%  B 2%  C 8%  D 33%  E 47%

**Question 8.**

From a pile of CDs, 14 of which are by rock artists, 6 by jazz artists, and 12 by country artists, two CDs are taken out at random and played, one at a time, without replacement. Draw a possibility tree and use it to answer the following questions.

   a.) What is the probability that a rock CD is played first, followed by a country CD?
   b.) What is the probability that both CDs are by the same kind of artist?

Outcomes assessed: Elementary probability

Degree of success: A 34%  B 6%  C 15%  D 19%  E 26%

**Question 9.**

Suppose that Michael borrows $230,000 for a new house. He signs a 25-year mortgage at 6.15% compounded monthly.

   a.) What is Michael’s monthly payment, to the nearest cent?
   b.) How much total interest will Michael pay over the 25-year period?
   c.) What is the unpaid balance of the loan after 12 years?

Outcomes assessed: Mathematics of finance

Degree of success: A 12%  B 9%  C 46%  D 21%  E 12%

**Question 10.**

It has been found that there is a linear relationship between the length, \(x\), of the femur (thigh bone), and the height, \(y\), of the human from whom it came. Suppose that it is known that a femur of length 47.5 cm corresponds to a height of 177.8 cm and that a femur of length 39.1 cm corresponds to a height of 146.3 cm.

   a.) Write a formula which gives the height in terms of the length of the femur.
   b.) Rounding to two decimal places, what height would correspond to a femur of length 52 cm?

Outcomes assessed: Systems of linear equations

Degree of success: A 21%  B 4%  C 19%  D 27%  E 29%

**Question 11.**

The officers of a high school senior class are planning to rent buses and vans for a class trip. Each bus can transport 40 students, requires 3 chaperones, and costs $1200 to rent.
Each van can transport 8 students, requires 1 chaperone, and costs $100 to rent. The officers must plan to accommodate at least 400 students. Since only 36 parents have volunteered to serve as chaperones, the officers must plan to use at most 36 chaperones. Linear programming can be used to determine how many of each type of vehicle to rent in order to minimize the transportation costs.

a.) Define variables and write the objective function for this problem.
b.) Write all of the constraints.

Outcomes assessed: Systems of linear inequalities

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<tr>
<th>Degree of success</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<td>9%</td>
<td>15%</td>
<td>15%</td>
<td>27%</td>
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**Question 12. (Bonus)**

Jessica and Jon are at a dinner party. There are 12 people (including Jessica and Jon) at the party sitting around a large round table. People were assigned a seat at random. What is the probability that Jessica and Jon are sitting next to each other?

Outcomes assessed: Elementary probability, combinations and permutations

<table>
<thead>
<tr>
<th>Degree of success</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<tbody>
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<td></td>
<td>1%</td>
<td>0%</td>
<td>10%</td>
<td>11%</td>
<td>78%</td>
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</tbody>
</table>

**III.3.2. Reflection.**

1. What did you learn from these data?

Instructor 1. Honestly, I did not find the data very helpful for a few reasons. First, the section that my data fell under is a multiple-instructor section. It may have been more helpful to see how the scores broke down by instructor, since grading scales can vary by person. Second, the scale by which one obtains a B, C, D, or E category score was not clear. The rating scale makes sense, but there was no mention of how one falls into the 80-99 category, i.e. what the typical student does to receive such a score. Third, you only took into account the final exam scores, but do not mention how the final exams compare to the rest of the semester. This fails to establish a comparison base for how a particular instructor grades over the course of the whole semester. Are there curves for some instructors? Do they drop points or take the exam out of a lesser number of possible points? It leaves a lot in question in order to make an accurate assessment of what changes should be made.

Instructor 2. I thought the results were interesting as far as the breakdown of how my students did on each problem on the final exam. But they weren’t very surprising. There was really only one question that students didn’t do as well as I would’ve predicted. And the success rate was the worst on the material that I consider to be the most difficult.

Instructor 3. This data was able to show us, the math department, that the students in Math 104 take a final exam each semester that results in mixed messages on the success
of each topic taught. It may be that the final exam week creates extra stress and/or pressure on the student or that they are not mastering the topics as well as we may like them to. The specific topics I see room for improvement are the Cost/Profit/Revenue graphs and questions about the graphs and Higher Level probability questions such as multiple events and the use of permutations and combinations. They seem do perform well with basic linear graphs and systems of equations but also struggle with setting up linear programming problems. I learned from this data that there is always room for improvement on the part of the instructors methods as well as the students time and effort on homework and study skills - set forth each week throughout the semester. Consistent practice and a sense of openness are key to the success of the student. Comfortable communication practices (not afraid to ask a question) and a regular homework collection practice is essential to keeping good work ethic among the classroom. This, in turn, should lead to productivity and learning in each class.

2. What will you do differently as a result of what you learned?

Instructor 1. I am unsure what changes should be made according to the assessment. Since I am one of the few who have only a fraction of a single data set, it is unclear which changes would need to be made on my part to affect the students in a positive manner.

Instructor 2. I don’t think I’ll do much differently based on the results.

Instructor 3. As an instructor, I will collect homework more often (every class period - versus once a week). I will encourage the use of a study group on a regular basis (outside of class). Finally, I will share my findings and ideas with my peers in the math department to encourage improvement within all of us. In conclusion, I feel that consistent homework collection, study groups, along with the present activities of an open communication style, an enjoyable classroom experience and the mutual sharing among peers in the math department are the key ingredients to success in learning math at NDSU.
III.4. Math 104, Finite Mathematics, fourth section

III.4.1. Final exam.

Question 1.
Write the equation of the dashed line on the graph. Are solid and dashed lines parallel? Explain your answer.

Outcomes assessed: N/A
Degree of success: A 65%  B 8%  C 10%  D 2%  E 15%

Question 2.
Let $f(x) = \sqrt{1 + 2x} + \frac{5}{x^2 - x - 2}$ and $g(x) = 7x^2 + 23x - 20$.

a.) Find the domain of $f$.
b.) Find the intercepts of $g$.
c.) Find $f(4) - 2g(6t + 3)$. (You do not need to simplify your answer.)

Outcomes assessed: N/A
Degree of success: A 20%  B 15%  C 13%  D 15%  E 37%

Question 3.
When Bridget takes a new job, she is offered an $1800 bonus now or the option of an extra $100 bonus each month for the next year and a half. She plans to invest her bonus money in an account that earns 6% interest compounded monthly. Which choice will earn her the most money at the end of the 18 months?

Outcomes assessed: Mathematics of finance
Degree of success: A 45%  B 3%  C 0%  D 0%  E 52%
Question 4.
A loan of $13,800 is to be repaid with weekly payments for 25 years at 8.35% interest compounded weekly. Calculate the weekly payment

Outcomes assessed: Mathematics of finance

Degree of success: A 68%  B 12%  C 5%  D 3%  E 12%

Question 5.

Let \( A = \begin{bmatrix} 2 & x & 3 \\ 0 & y & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix} \) and \( C = \begin{bmatrix} 3 & -2 \\ 0 & -1 \\ 1 & 2 \end{bmatrix} \).

Find \( AC - 2B \).

Outcomes assessed: Matrices

Degree of success: A 63%  B 15%  C 2%  D 0%  E 20%

Question 6.

Solve the following problem without using a calculator

A group of sociologists have grant money to study school busing in Fargo. They wish to conduct an opinion survey using 780 telephone surveys, 760 house surveys, and 940 online surveys. They hire three companies to conduct the surveys. Company A has personnel to do 30 telephone, 10 house, and 25 online surveys per hour. Company B has personnel to do 20 telephone, 20 house, and 30 online surveys per hour. Company C has personnel to do 15 telephone, 30 house, and 20 online surveys per hour. How many hours should be scheduled for each company to produce exactly the number of surveys needed?

Outcomes assessed: Systems of linear equations

Degree of success: A 45%  B 5%  C 3%  D 0%  E 47%

Question 7.

Infotron Inc. makes electronic hockey and soccer games. Each hockey game requires 2 labor-hours of assembly and 2 labor-hours of testing. Each soccer game requires 3 labor-hours of assembly and 1 labor-hour of testing. Each day there are 70 labor-hours available for assembly and 42 labor-hours available for testing. How many of each game should Infotron produce each day to maximize its total output?

Outcomes assessed: Systems of linear inequalities, linear programming

Degree of success: A 60%  B 15%  C 5%  D 0%  E 20%
Question 8.

An election ballot asks voters to select a mayor, assistant mayor, city planner, and four city commissioners from a group of 20 candidates. Assuming that no person can hold more than one position, how many ways can this be done?

Outcomes assessed: Elementary probability
Degree of success: A 10%  B 3%  C 2%  D 0%  E 85%

Question 9.

A basket contains 7 red marbles and 9 green marbles. One marble is drawn, its color noted and placed back into the basket. A second marble is then drawn and its color is noted.

a.) What is the probability that exactly one red marble is drawn?
b.) What are the odds that the first marble is green?

Outcomes assessed: Elementary probability
Degree of success: A 50%  B 3%  C 12%  D 8%  E 27%

Question 10.

Use the table below to answer the following questions.

| 3 5 6 7 7 8 10 12 17 19 |
| 4 5 6 7 7 9 11 12 17 22 |
| 4 6 6 7 7 9 11 13 17 24 |
| 4 6 6 7 8 9 11 13 17 28 |
| 5 6 6 7 8 10 12 15 18 29 |
| 5 6 6 7 8 10 12 16 18 33 |

a.) Find the median and mode(s).
b.) Identify any outliers.
c.) What is the percentile for 17?

Outcomes assessed: Descriptive statistics
Degree of success: A 10%  B 12%  C 2%  D 13%  E 63%

Question 11.

Using the data set below, find the z-score for 27.

```
19 26 27 34 35
```

Outcomes assessed: Descriptive statistics
Degree of success: A 15%  B 0%  C 7%  D 3%  E 75%
III.4.2. **Reflection.**

1. What did you learn from these data?

I learned that the students did not do so well on the material that was presented during the last several days of the semester.

2. What will you do differently as a result of what you learned?

I would give additional graded assignments on this material to encourage the students to learn these topics.

III.4.3. **Formative Assessment.** I give weekly quizzes based on the suggested homework problems. I also do several examples during class. I often have the students work with their neighbor to solve the problem before going over the problem on the board. I also have the students walk me through how to solve each problem.
III.5. Math 128, Introduction to Linear Algebra, first section

III.5.1. Final exam.

Question 1.

Find all numbers $x$ such that

\[
\begin{vmatrix}
  x + 1 & 1 & 2 & x^4 \\
  0 & 2 & 3 & x^3 \\
  0 & 0 & x - 1 & x^2 \\
  0 & 0 & 0 & 1
\end{vmatrix} = \begin{vmatrix}
  1 - x & 9 & 0 \\
  4 & 1 - 2x & -2 \\
  2 & 3 & 1
\end{vmatrix}.
\]

Outcomes assessed: Row operations, echelon form, determinants

Degree of success: A 27%  B 31%  C 23%  D 13%  E 6%

Question 2.

Let $A$, $B$, and $C$ be invertible $4 \times 4$ matrices such that $\det(A) = 4$, $\det(B) = -7$, $\det(C) = 15$.

a.) Find $\det(2A)$.

b.) Find $\det(B^2C^{-1})$.

c.) Find $\det((AB)^T)$.

Outcomes assessed: Matrix operations, inverses, determinants

Degree of success: A 37%  B 6%  C 36%  D 19%  E 2%

Question 3.

Let $A = \begin{pmatrix} 2 & 5 & 3 \\ 1 & 2 & 7 \\ 3 & 6 & 0 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix}$.

Use Cramer’s Rule to find $x_2$ in the system $A\vec{x} = \vec{b}$.

Outcomes assessed: Systems of linear equations, determinants

Degree of success: A 63%  B 19%  C 6%  D 10%  E 2%

Question 4.

For parts a) and b) below, determine whether the described matrix exists. If so, give an example and explain why your example works. If not, explain specifically why no such example can exist.

a.) $A$ is a $3 \times 3$ invertible matrix such that $A^2 = 3A$.

b.) $B$ is an $n \times n$ matrix such that $B^T$ is invertible and the columns of $B$ are linearly dependent.
Outcomes assessed: Matrix operations, inverses, determinants
Degree of success: A 10%  B 6%  C 31%  D 51%  E 2%

**Question 5.**

Let \( A = \begin{pmatrix} 2 & -3 & 3 \\ -6 & 9 & -7 \\ 0 & 3 & 2 \end{pmatrix} \). Find \( A^{-1} \). Show all work.

Outcomes assessed: Row operations, echelon form, inverses
Degree of success: A 52%  B 23%  C 8%  D 17%  E 0%

**Question 6. (Bonus)**

How many matrices of the form
\[
\begin{pmatrix} 2 & a \\ b & 3 \end{pmatrix}
\]
are not invertible if \( a \) and \( b \) are both integers?

Outcomes assessed: Inverses, determinants
Degree of success: A 38%  B 10%  C 10%  D 25%  E 17%

**III.5.2. Reflection.**

1. What did you learn from these data?

Instructor 1. I learned that my students appear to be learning certain topics better than others.

Instructor 2. The data regarding the exams is both interesting but not surprising. It is not surprising because I think about the problems I put on exams beforehand. Generally, based on student performance on quizzes and conversations with students during office hours I know how the exam is going to go. There are questions that I put on every exam I write (as I’ve taught this course multiple times) that repeatedly give students trouble. Even when I give multiple examples and explanations of an idea, some problems still give students difficulty. I don’t think the problem is that these problems are too challenging, as I try to keep most of the exam in line with problems that are given in the text.

The data is interesting because it confirms some things I have thought, but being confronted with it, makes me rethink what I do instructionally. For instance, certain problems have distributions with large numbers of students grouped in two categories, with few students in the category between the categories. I wonder if the flaw is in the question, the grading, or if there is no flaw. Many questions do tend to be contingent on one central idea. If a student does not understand that idea, the score will obviously be lower. However, I try to avoid these questions and break things up into smaller parts when necessary, but looking at the data it seems it still happens.
2. What will you do differently as a result of what you learned?

Instructor 1. I would try to focus on some of the problem areas that my students are struggling with specifically by setting aside a lecture each month to target these problem areas in depth.

Instructor 2. I'm not sure I'm going to change anything. As I said, I think about what I do every time I give an exam and teach. Teaching is an evolving process that I go through every day of a semester, and this data has not really changed what I think of teaching linear algebra.

III.5.3. Formative Assessment. Instructor 2. I use quiz results to gauge student understanding. I quiz on material within one week of the students’ first exposure to topic. Based on those results, I may review an idea that was not grasped.

When students attend my office hours I will ask them how they are thinking about a problem, or how their understanding has changed from when they came in for help and when they leave.
III.6. Math 128, Introduction to Linear Algebra, second section

III.6.1. Final exam.

**Question 1.**

Let \( A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix} \). What value(s) of \( k \), if any, will make \( AB = BA \)?

Outcomes assessed: Matrix operations

Degree of success: A 69%  B 7%  C 7%  D 10%  E 7%

**Question 2.**

Find a pair of \( 2 \times 2 \) matrices \( A \) and \( B \) with all nonzero entries such that \( AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \).

Show clearly that your example is valid.

Outcomes assessed: Matrix operations

Degree of success: A 86%  B 0%  C 3%  D 11%  E 0%

**Question 3.**

Use a cofactor expansion down the third column to calculate

\[
\begin{vmatrix} 2 & -3 & 4 \\ 0 & -1 & 6 \\ -3 & 2 & -5 \end{vmatrix}.
\]

Outcomes assessed: Determinants

Degree of success: A 59%  B 10%  C 17%  D 14%  E 0%

**Question 4.**

Suppose that \( A \) is a \( 9 \times 9 \) matrix and that \( B \) is obtained from \( A \) by the following sequence of row operations:

\[
2R_1 + R_8 \rightarrow R_8, -3R_2 \rightarrow R_2, R_3 \leftrightarrow R_4, -1.56R_2 + R_5 \rightarrow R_5, 2.4R_6 \rightarrow R_6
\]

If \( \det B = 6 \), calculate \( \det A \).

Outcomes assessed: Determinants

Degree of success: A 66%  B 10%  C 21%  D 3%  E 0%

**Question 5.**

Some guy wearing a trenchcoat and dark sunglasses told me that if \( A \) and \( B \) are invertible \( n \times n \) matrices, then \( A + B \) is also invertible. Was this advice correct? Either explain why or give a specific counterexample to demonstrate why not.

Outcomes assessed: Inverses
Degree of success: A 69%  B 0%  C 0%  D 24%  E 7%

Question 6.

Find all nonzero values of $x$ and $y$ so that the matrix
\[
\begin{bmatrix}
x & 0 & y \\
3 & 2 & 2 \\
1 & 1 & 6
\end{bmatrix}
\]
is not invertible.

Outcomes assessed: Inverses, determinants

Degree of success: A 59%  B 17%  C 7%  D 7%  E 10%

Question 7.

Let $A$ be a $3 \times 3$ matrix and suppose that $\det A = -7$.

a.) Calculate $\det(4A^2)$.

b.) Calculate $\det(2A^{-1})$.

c.) Calculate $\det((2A)^{-1})$.

Outcomes assessed: Determinants

Degree of success: A 69%  B 4%  C 24%  D 0%  E 3%

Question 8.

Choose from the following list all of the statements that are true (list the letters).

a.) If $A$ is a $3 \times 3$ matrix with linearly independent columns, then the columns of $A$ span $\mathbb{R}^3$.

b.) If $u$ and $v$ are solutions to the homogeneous matrix equation $Ax = 0$, then $u + v$ is also a solution to that equation.

c.) If $A$ is an $n \times n$ matrix that is not invertible, then the matrix equation $Ax = b$ has an infinite number of solutions for every $b$ in $\mathbb{R}^n$.

d.) If $A$ is a square matrix with two identical columns, then $\det A = 0$.

e.) If $A$ and $B$ are $n \times n$ matrices and $B$ is not invertible, then $AB$ will not be invertible.

f.) If $A$ is an invertible $n \times n$ matrix, then $AA^T = I_n$.

Outcomes assessed: Systems of linear equations, echelon form, matrix operations, inverses, determinants

Degree of success: A 3%  B 0%  C 55%  D 41%  E 0%

Question 9. (Bonus)

Suppose that $A$ and $B$ are invertible $n \times n$ matrices. Express the inverse of $(AB)^2$ in terms of $A^{-1}$ and $B^{-1}$.

Outcomes assessed: Matrix operations, inverses
III.6.2. **Reflection.**

1. What did you learn from these data?

As suspected, students tend to do very well in this class with problems that involve mostly mechanics and struggle a bit with more indirect questions that involve a deeper understanding of terminology or visualizing examples that may support or disprove a statement.

2. What will you do differently as a result of what you learned?

I would just continue to regularly ask the students on assignments, quizzes, etc. to answer questions that are more conceptual in nature so that they become more accustomed to looking at material in a way other than just how to apply algorithms to solve certain types of problems.

III.6.3. **Formative Assessment.** I have three items that I think are appropriate.

i) I will have students, assembled in small groups, compile a list of "facts" about some particular topic (lines, for instance). These lists will be collected and redistributed so that each group has a list other than the one they had produced. We will then go around the class, group by group, and have each group report one "fact" that they feel is worth recording on the board. This will be repeated until either groups are unable to come up with new things to record or space runs out. Then, we will discuss, of all of the "facts", which seem to be the ones that would be most useful in terms of the types of topics we would be studying in a calculus course.

ii) I will have students individually work on a problem related to new material (that hasn’t been explored in class yet), and collect their work. These papers will then be redistributed to the class, so that nobody has their own paper returned to them. I will then have them assemble into small groups and consider the papers they have been given. Each group then decides on which of the papers has the "best" solution, and will propose that solution (as well as explain the solution, as best they can), and I will record the final conclusion on the board. Once every group has made a proposal, we will discuss the similarities and differences between approaches and the students are invited to form an opinion on whether or not a particular proposal seems reasonable (without calling attention to the person that actually provided the solution). One example of such an activity is that I will provide a table with periodic velocity measurements for a car moving along a straight path and ask the students to estimate the distance the car travels over a specific interval of time, as an introduction to definite integrals and the associated sums.
iii) I will have students work in small groups on some problem(s) related to recently covered material and have a representative of each group present (on the board) a proposal for solution to a problem or part of solution to a problem and solicit feedback from the rest of the class pertaining to agreement/disagreement with some part of the proposal. Each group will be asked to contribute something on the board, whether it be a complete solution to a small problem or part of a solution to a larger problem.
III.7. Math 128, Introduction to Linear Algebra, Third Section

III.7.1. Final Exam.

Question 1.

Let \( A = \begin{bmatrix} 1 & 4 & 5 \\ 7 & 8 & \ln(h) \\ -2 & 1 & 3 \end{bmatrix} \). Find all values of \( h \) such that \( A^T \) is invertible. (Hint: Use the invertible matrix theorem.)

Outcomes assessed: Inverses

Degree of success: A 32%  B 36%  C 32%  D 0%  E 0%

Question 2.

Let \( A \) and \( B \) be 4 \times 4 matrices such that \( \det B = -14 \) and \( B \) is produced by performing the following sequence of operations on \( A \):

\[
3R_2 \rightarrow R_2, 6R_1 + R_2 \rightarrow R_2, R_1 \leftrightarrow R_3, -2R_2 + R_4 \rightarrow R_4, -2R_4 \rightarrow R_4
\]

a.) Find \( \det A \).

b.) Find \( \det((4AB)^{-1}) \).

c.) Find \( \det(2AB^T) \).

d.) Find \( \det(2B + 3I_4B) \).

e.) Find \( \det(B^{-1}A^2B) \).

Outcomes assessed: Determinants

Degree of success: A 10%  B 58%  C 16%  D 13%  E 3%

Question 3.

Let \( A = \begin{bmatrix} a & b & c & d & 0 \\ e & 0 & f & g & 0 \\ h & i & j & k & z \\ m & 0 & 0 & 0 & 0 \\ n & 0 & p & q & 0 \end{bmatrix} \). Without the use of a calculator, find \( \det A \).

Outcomes assessed: Determinants

Degree of success: A 81%  B 13%  C 6%  D 0%  E 0%

Question 4.

Let \( A = \begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix} \), \( D = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \) and \( C = \begin{bmatrix} 2 & 7 & 5 \\ 3 & 2 & 12 \end{bmatrix} \). Find a matrix \( B \) such that \( DAB = C \). Explain.

Outcomes assessed: Matrix operations
Degree of success: A 52%  B 29%  C 6%  D 13%  E 0%

**Question 5.**

An $n \times n$ matrix $C$ is called **symmetric** if $C = C^T$. Prove that if $A, B$ are symmetric matrices, then $AB$ is symmetric if and only if $AB = BA$. Clearly justify your proof.

Outcomes assessed: Matrix operations

Degree of success: A 0%  B 0%  C 3%  D 7%  E 90%

**Question 6. (Bonus)**

Show that if $A$ is an $n \times n$ triangular matrix, then $\det A$ is the product of the entries on the main diagonal of $A$. Clearly explain why.

Outcomes assessed: Determinants

Degree of success: A 16%  B 0%  C 0%  D 0%  E 84%

**III.7.2. Reflection.**

1. What did you learn from these data?

I notice, the students are struggling with some particular topics in linear algebra such as Vector spaces, dimension of the kernel, bases, matrix operations, linearly independence.

2. What will you do differently as a result of what you learned?

If I teach linear algebra again I probably will spend more time covering these particular topics, show them more examples.
III.8. Math 128, Introduction to Linear Algebra, fourth section

III.8.1. Final exam.

Question 1.

Let \[ B = \begin{bmatrix} a + b & c & 1 \\ b + c & a & 1 \\ c + a & b & 1 \end{bmatrix} \]. Determine all possible values of \( a, b \) and \( c \), if any, so that matrix \( B \) is invertible.

Outcomes assessed: Inverses

Degree of success: A 33%  B 6%  C 0%  D 30%  E 31%

Question 2.

Let \[ A = \begin{bmatrix} 1 & 0 & 4 \\ -3 & x & -6 \\ 1 & 3 & 2-x \end{bmatrix} \] and \[ B = \begin{bmatrix} x & 2 \\ 3 & 1-x \end{bmatrix} \]. Find all values of \( x \) such that \( \det A = \det B \).

Outcomes assessed: Determinants

Degree of success: A 55%  B 36%  C 3%  D 6%  E 0%

Question 3.

Let \( A \) and \( B \) be \( 3 \times 3 \) matrices such that \( \det A = 3 \) and \( B \) is produced by performing the following sequence of operations on \( A \):

\[ R_1 \leftrightarrow R_3, -2R_1 \to R_1, 4R_2 + R_3 \to R_3, \sqrt{3} R_1 \to R_1, 2R_3 + R_1 \to R_1, \frac{1}{2} R_2 \to R_2, 2R_3 + 5R_1 \to R_1 \]

a.) Find \( \det B \).

b.) Find \( \det(\sqrt{3}(A^{-1})^3B^T B^2) \).

c.) \( \det(B A + 2B(AB)^T B A I_3(BA)^{-1}((AB)^T)^{-1})A) \).

Outcomes assessed: Determinants

Degree of success: A 15%  B 21%  C 33%  D 19%  E 12%

Question 4.

Determine if the following statements are always true, sometimes true, or never true. No justification is necessary.

a.) If \( \det(AB) = 0 \), then \( \det A = 0 \) or \( \det B = 0 \).

b.) If \( A \) and \( B \) are invertible \( n \times n \) matrices, then \( A + B \) is also invertible.

c.) If a \( 7 \times 7 \) matrix \( K \) has 5 pivot rows, then \( K \) is row equivalent to \( I_7 \).

d.) If \( A, B, C \) and \( D \) are invertible matrices, then \( ((AB)^{-1}(CD)^{-1}))^{-1} = ABCD \).
e.) Suppose $A$ is an $n \times n$ matrix, if the equation $A\vec{x} = \vec{b}$ has a solution for each $\vec{b}$ in $\mathbb{R}^n$, then $A$ is invertible.

Outcomes assessed: Systems of linear equations, echelon form, matrix operations, inverses, determinants

Degree of success: A 15%  B 40%  C 0%  D 27%  E 18%

**Question 5.**

Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What values of $k$, if any, will make $AB = BA$? Show your work.

Outcomes assessed: Matrix operations

Degree of success: A 49%  B 15%  C 9%  D 6%  E 21%

**Question 6. (Bonus)**

If $A$ is an $n \times n$ matrix, then $A$ is called **idempotent** if $A^2 = A$.

a.) Show that $AB$ is idempotent if $AB = BA$.

b.) Show that if $A$ is idempotent, then $A^T$ is idempotent.

Outcomes assessed: Matrix operations

Degree of success: A 3%  B 0%  C 0%  D 9%  E 88%

**III.8.2. Reflection.**

1. What did you learn from these data?

I notice, the students are struggling with some particular topics in linear algebra such as Vector spaces, dimension of the kernel, bases, matrix operations, linearly independence.

2. What will you do differently as a result of what you learned?

If I teach linear algebra again I probably will spend more time covering these particular topics, show them more examples.
III.9. Math 129, Basic Linear Algebra, first section

III.9.1. Final exam.

**Question 1.**

Determine if the following statements are true or false. Justify your response.

a.) There exists a system of 4 linear equations with four variables and exactly four solutions.

b.) If $\vec{u}$ and $\vec{v}$ are solutions to the matrix equation $A\vec{x} = \vec{b}$, then $3\vec{u} - 2\vec{v}$ is also a solution.

Outcomes assessed: Systems of linear equations, matrix operations, linear independence

Degree of success: A 27%  B 17%  C 13%  D 33%  E 10%

**Question 2.**

Find the standard matrix of the linear transformation $T$ defined by

$$T\left( \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ -1 \\ -7 \end{bmatrix} \quad \text{and} \quad T\left( \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right) = \begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix}.$$ 

Outcomes assessed: Systems of linear equations, row operations, echelon form

Degree of success: A 57%  B 10%  C 3%  D 17%  E 13%

**Question 3.**

Let $M_{2\times2}$ be the set of all $2 \times 2$ matrices with real entries. Given that $M_{2\times2}$ is a vector space, indicate whether each of the following is a subspace of $M_{2\times2}$. No justification is necessary.

a.) The set of all matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $a, b, c$ and $d$ are real numbers with $a + b + c + d > 0$.

b.) The set of all matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $a, b, c$ and $d$ are real numbers with $a + b + c + d = 0$.

c.) The set of all matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $a, b, c$ and $d$ are real numbers with $abcd \geq 0$.

d.) The set of all $2 \times 2$ matrices $A$ such that $\det A = 0$.

e.) The set of all $2 \times 2$ matrices $A$ such that $\begin{bmatrix} 2 & 1 \\ 7 & 3 \end{bmatrix} A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Outcomes assessed: Subspaces
Degree of success: A 30%  B 20%  C 23%  D 27%  E 0%

Question 4.

Let \( A = \begin{bmatrix} 1 & 0 & 3 & -1 \\ 3 & 1 & 10 & -1 \\ 2 & 1 & 7 & 0 \end{bmatrix} \).

a.) Find a basis for Row \( A \).

b.) Find \( \dim(\text{Row}(A)) \).

c.) Find \( \dim(\text{Nul}(A)) \).

d.) Find \( \dim(\text{Col}(A)) \).

Outcomes assessed: Row operations, echelon form, vectors in \( n \)-space, subspaces, dimension

Degree of success: A 57%  B 20%  C 3%  D 20%  E 0%

Question 5.

Let \( A = \begin{bmatrix} 3 & 6 & -3 & 9 \\ -2 & -4 & 5 & -9 \end{bmatrix} \).

a.) Find a basis for Nul \( A \).

b.) Find \( \text{rank}(A) \).

Outcomes assessed: Systems of linear equations, row operations, echelon form, vectors in \( n \)-space, subspaces, homogeneous systems, rank, dimension

Degree of success: A 57%  B 20%  C 10%  D 10%  E 3%

Question 6.

Let \( H = \text{Span}\{x^3 + x^2, x^2 + x, x^3 + 2x^2 + x, x^3 + x^2 + x + 1\} \).

a.) Find a basis for \( H \).

b.) Find \( \dim(H) \).

Outcomes assessed: Linear independence, dimension

Degree of success: A 10%  B 30%  C 13%  D 40%  E 7%

Question 7.

Indicate whether each of the following statements is true or false. If the statement is true, justify your answer with an example.

a.) There is a vector space \( V \) that has a subspace \( S \) such that \( \dim(S) < \dim(V) \).

b.) There is a vector space \( V \) that has a subspace \( S \) such that \( \dim(S) > \dim(V) \).

c.) There is a vector space \( V \) that has a subspace \( S \) such that \( \dim(S) = \dim(V) \).
Outcomes assessed: Subspaces, dimension
Degree of success: A 10%  B 44%  C 23%  D 23%  E 0%

**Question 8.**

Let \( A = \begin{bmatrix} 0 & -2 & 0 \\ -1 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}. \)

a.) Find the eigenvalues of \( A \).
b.) Find a basis for the eigenspace associated to one of the eigenvalues of \( A \). (You may choose which eigenvalue to use.)

Outcomes assessed: Determinants, subspaces, linear independence
Degree of success: A 37%  B 17%  C 23%  D 20%  E 3%

**Question 9.**

Let \( A \) be an \( n \times n \) matrix with \( \det(A) = 0 \). Indicate whether each statement is always, sometimes, or never true. No justification is necessary.

a.) \( \text{rank}(A) = n - 1 \)
b.) \( \dim(\text{Nul}(A)) = 0 \)
c.) \( \text{Col}(A) = \mathbb{R}^n \)
d.) \( \dim(\text{Row}(A)) \neq n \)
e.) \( \dim(\text{Col}(A^T)) = 1 \)

Outcomes assessed: Matrix operations, vectors in \( n \)-space, subspaces, rank, dimension
Degree of success: A 26%  B 17%  C 17%  D 40%  E 0%

**Question 10. (Bonus)**

Show, or explain why, if \( A \) is \( n \times n \) matrix and \( n \) is odd, then \( A \) has at least one real eigenvalue.

Outcomes assessed: Determinants
Degree of success: A 10%  B 3%  C 20%  D 27%  E 40%

III.9.2. **Reflection.**

1. What did you learn from these data?
I learned that my students appear to be learning certain topics better than others.

2. What will you do differently as a result of what you learned?
I would try to focus on some of the problem areas that my students are struggling with specifically by setting aside a lecture each month to target these problem areas in depth.
III.10. Math 129, Basic Linear Algebra, second section


Determine whether \(
\begin{pmatrix} 72 \\ 46 \\ -45 \end{pmatrix}
\) is a linear combination of
\(
\begin{pmatrix} 9 \\ 6 \\ -7 \end{pmatrix}, \begin{pmatrix} -9 \\ -5 \\ 3 \end{pmatrix} \) and \(
\begin{pmatrix} -9 \\ -4 \\ -1 \end{pmatrix}
\)

Outcomes assessed: Systems of linear equations, row operations, echelon form, vectors in \(n\)-space

Degree of success: A 19%  B 21%  C 24%  D 31%  E 5%

**Question 2.**

Let \(T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} x + ky \\ -y \end{bmatrix} \) be a linear transformation.

a.) Show that \(T\) is one-to-one for all real values of \(k\).

b.) Show that \(T^{-1} = T\).

Outcomes assessed: Row operations, inverses, linear independence

Degree of success: A 55%  B 14%  C 7%  D 17%  E 7%

**Question 3.**

Let \(S\) be the region in \(\mathbb{R}^2\) bounded by the line \(y = -3x + 6\), the \(x\)-axis, and the \(y\)-axis.

Find the area of \(T(S)\) where \(T\) is the linear transformation defined by
\(T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} 3x + 5y \\ 4x - 2y \end{bmatrix} \).

Outcomes assessed: Determinants

Degree of success: A 29%  B 26%  C 21%  D 24%  E 0%

**Question 4.**

Let \(A = \begin{bmatrix} 0 & 2 & 4 \\ 3 & 5 & 1 \\ 1 & 3 & 3 \\ -3 & -2 & 5 \end{bmatrix} \).

a.) Find a basis for Col(A).

b.) Find dim(Col(A)).

c.) Find dim(Nul(A)).

d.) Find dim(Row(A)).
Outcomes assessed: Row operations, echelon form, vectors in $n$-space, subspaces, linear independence, dimension

Degree of success: A 60%  B 22%  C 4%  D 14%  E 0%

**Question 5.**

Let $A = \begin{bmatrix} 2 & -5 \\ -4 & 3 \end{bmatrix}$.

a.) Find all eigenvalues for $A$.  
b.) Find bases for the eigenspace of $A$.

Outcomes assessed: Determinants, subspaces

Degree of success: A 48%  B 24%  C 14%  D 12%  E 2%

**Question 6.**

Let $A$ be an $n \times n$ invertible matrix. Indicate whether each statement is always, sometimes, or never true. No justification is necessary.

a.) $A \vec{e}_1 = \vec{0}$  
b.) $\text{rank}(A) = n$  
c.) The columns of $A$ are a basis for $\mathbb{R}^n$  
d.) The rows of $A$ are a basis for $\mathbb{R}^n$  
e.) $\det A = n$

Outcomes assessed: Matrix operations, inverses, determinants, vectors in $n$-space, subspaces, homogeneous systems, rank, dimension

Degree of success: A 24%  B 22%  C 12%  D 38%  E 4%

**Question 7.**

Find all values of $k$, if any, such that \{ $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix}$, $\begin{bmatrix} k & 2k \\ 4 & 5 \end{bmatrix}$ \} is a linearly independent set of vectors in $M_{2 \times 2}$.

Outcomes assessed: Matrix operations, inverses, determinants, linear independence

Degree of success: A 12%  B 40%  C 28%  D 20%  E 0%

**Question 8.**

Let $A = \begin{bmatrix} -1 & -3 & -6 & 1 & -1 \\ 2 & 6 & 6 & 1 & 5 \\ 1 & 3 & 6 & -1 & 1 \\ 2 & 6 & 12 & -2 & 9 \end{bmatrix}$.
Row operations are used to change $A$ into the matrix

$$
\begin{bmatrix}
1 & 3 & 0 & 2 & 4 \\
0 & 0 & 6 & -3 & 4 \\
0 & 0 & 0 & 0 & -7 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

a.) Find a basis for $\text{Col}(A)$.
b.) Find a basis for $\text{Nul}(A)$.

Outcomes assessed: Row operations, echelon form, vectors in $n$-space, subspaces, linear independence

Degree of success: A 31% B 15% C 33% D 19% E 2%

Question 9.
Indicate whether each of the following statements is true or false. If the statement is true, justify your answer with an example.

a.) There is a vector space $V$ that has a subspace $S$ such that $\dim(S) < \dim(V)$.
b.) There is a vector space $V$ that has a subspace $S$ such that $\dim(S) > \dim(V)$.
c.) There is a vector space $V$ that has a subspace $S$ such that $\dim(S) = \dim(V)$.

Outcomes assessed: Subspaces, dimension

Degree of success: A 26% B 24% C 17% D 28% E 5%

Question 10.
Find an example of an $n \times n$ matrix $A$ such that $\dim(\text{Nul}(A)) + \text{rank}(A) = \dim(\text{Nul}(A)) \cdot \text{rank}(A)$.

Outcomes assessed: Subspaces, rank, dimension

Degree of success: A 10% B 4% C 12% D 43% E 31%

III.10.2. Reflection.

1. What did you learn from these data?

Instructor 1. I learned that my students appear to be learning certain topics better than others.

Instructor 2. The data regarding the exams is both interesting but not surprising. It is not surprising because I think about the problems I put on exams beforehand. Generally, based on student performance on quizzes and conversations with students during office hours I know how the exam is going to go. There are questions that I put on every exam I write (as I've taught this course multiple times) that repeatedly give students trouble. Even when I give multiple examples and explanations of an idea, some problems still give
students difficulty. I don’t think the problem is that these problems are too challenging, as I try to keep most of the exam in line with problems that are given in the text.

The data is interesting because it confirms some things I have thought, but being confronted with it, makes me rethink what I do instructionally. For instance, certain problems have distributions with large numbers of students grouped in two categories, with few students in the category between the categories. I wonder if the flaw is in the question, the grading, or if there is no flaw. Many questions do tend to be contingent on one central idea. If a student does not understand that idea, the score will obviously be lower. However, I try to avoid these questions and break things up into smaller parts when necessary, but looking at the data it seems it still happens.

2. What will you do differently as a result of what you learned?

Instructor 1. I would try to focus on some of the problem areas that my students are struggling with specifically by setting aside a lecture each month to target these problem areas in depth.

Instructor 2. I’m not sure I’m going to change anything. As I said, I think about what I do every time I give an exam and teach. Teaching is an evolving process that I go through every day of a semester, and this data has not really changed what I think of teaching linear algebra.

III.10.3. Formative Assessment. Instructor 2. I use quiz results to gauge student understanding. I quiz on material within one week of the students’ first exposure to topic. Based on those results, I may review an idea that was not grasped.

When students attend my office hours I will ask them how they are thinking about a problem, or how their understanding has changed from when they came in for help and when they leave.
III. Math 129, Basic Linear Algebra, third section

III.11. Final exam.

Question 1.

Let \( A = \begin{pmatrix} 6 & -2 & 0 & 5 \\ 12 & -2 & 3 & -6 \\ -6 & 8 & 9 & 17 \end{pmatrix} \).

a.) Describe all solutions to \( A \vec{x} = \vec{0} \) in parametric vector form.

b.) Suppose \( A \begin{pmatrix} 2 \\ -1 \\ 6 \\ 8 \end{pmatrix} = \vec{b} \). Describe all solutions to \( A \vec{x} = \vec{b} \) in parametric vector form.

Outcomes assessed: Systems of linear equations, row operations, echelon form, homogeneous systems

Degree of success: A 3% B 0% C 43% D 54% E 0%

Question 2.

Let \( A = \begin{bmatrix} -7 & 5 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \) and \( C = \begin{bmatrix} 2 & 2 \\ 3 & 2 \end{bmatrix} \). Find \((A(BC)^{-1})^{-1}\).

Outcomes assessed: Matrix operations, inverses

Degree of success: A 46% B 0% C 36% D 18% E 0%

Question 3.

Let \( A \) be a 4 \times 4 matrix and \( \det(A) = 4 \). The matrix \( B \) is obtained by performing the following row operations to \( A \):

\[ R_1 \leftrightarrow R_2, \, 2R_2 + R_3 \rightarrow R_3, \, -2R_4 \rightarrow R_4, \, 3R_3 + R_1 \rightarrow R_1, \, 3R_1 \rightarrow R_1 \]

a.) Find \( \det(B) \).

b.) Find \( \det((AB)^T)^{-1} \).

c.) Find \( \det(AB - 3AB^2I_4B^{-1}) \).

Outcomes assessed: Row operations, determinants

Degree of success: A 25% B 14% C 32% D 29% E 0%
Question 4.

Let \( A = \begin{bmatrix} -2 & 6 & 0 \\ 3 & -9 & 3 \\ 5 & -15 & 2 \\ -1 & 3 & 3 \end{bmatrix} \).

a.) Find a basis for \( \text{Row}(A) \).
b.) Find \( \text{dim}(\text{Row}(A)) \).
c.) Find \( \text{dim}(\text{Nul}(A)) \).
d.) Find \( \text{rank}(A) \).

Outcomes assessed: Row operations, echelon form, vectors in \( n \)-space, subspaces, rank, dimension

Degree of success: A 53%  B 25%  C 11%  D 11%  E 0%

Question 5.

Let \( A = \begin{bmatrix} 6 & 3 \\ 2 & 7 \end{bmatrix} \).

a.) Find all eigenvalues for \( A \).
b.) Find bases for the eigenspaces of \( A \).

Outcomes assessed: Eigenvalues, eigenspaces

Degree of success: A 43%  B 29%  C 0%  D 28%  E 0%

Question 6.

Let \( H = \text{Span}\left\{ \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & -7 \\ -2 & 2 \end{bmatrix}, \begin{bmatrix} -1 & -14 \\ -1 & 8 \end{bmatrix} \right\} \). Find a basis for \( H \).

Outcomes assessed: Linear independence, row operations, subspaces

Degree of success: A 54%  B 11%  C 14%  D 14%  E 7%

Question 7.

Define \( T : M_{2\times2} \rightarrow \mathbb{P}_2 \) by \( T \left( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = (a - b)x^2 + (2b + c)x + (a + b + c) \). Find a basis for \( \text{Ker} T \).

Outcomes assessed: Subspaces, homogeneous systems

Degree of success: A 11%  B 14%  C 21%  D 50%  E 4%

Question 8.

Let \( A \) be an \( n \times n \) invertible matrix. Indicate whether each statement is always, sometimes, or never true. No justification is necessary.
a.) $A \vec{e}_1 = \vec{0}$
b.) $\text{rank}(A) = n$
c.) The columns of $A$ are a basis for $\mathbb{R}^n$
d.) The rows of $A$ are a basis for $\mathbb{R}^n$
e.) $\det(A) = n$

Outcomes assessed: Matrix operations, inverses, determinants, rank

Degree of success: A 43%  B 29%  C 14%  D 11%  E 3%

**Question 9.**

If $V$ is an infinite dimensional vector space, show, or explain why, there is a subspace $H$ of $V$ with $\text{dim}(H) = 317$.

Outcomes assessed: Subspaces

Degree of success: A 0%  B 4%  C 18%  D 68%  E 10%

**Question 10.** *(Bonus)*

Let $F$ be the vector space of all functions $f : \mathbb{R} \rightarrow \mathbb{R}$. Determine whether each of the following sets is linearly independent in $F$. Clearly justify your answer.

a.) $\{\sin 2x, \sin x \cos x\}$
b.) $\{x^2, 2^x\}$

Outcomes assessed: Linear independence

Degree of success: A 39%  B 0%  C 21%  D 25%  E 14%

**III.11.2. Reflection.**

1. What did you learn from these data?

The data regarding the exams is both interesting but not surprising. It is not surprising because I think about the problems I put on exams beforehand. Generally, based on student performance on quizzes and conversations with students during office hours I know how the exam is going to go. There are questions that I put on every exam I write (as I’ve taught this course multiple times) that repeatedly give students trouble. Even when I give multiple examples and explanations of an idea, some problems still give students difficulty. I don’t think the problem is that these problems are too challenging, as I try to keep most of the exam in line with problems that are given in the text.

The data is interesting because it confirms some things I have thought, but being confronted with it, makes me rethink what I do instructionally. For instance, certain problems have distributions with large numbers of students grouped in two categories, with few students in the category between the categories. I wonder if the flaw is in the question, the grading, or if there is no flaw. Many questions do tend to be contingent on one central
idea. If a student does not understand that idea, the score will obviously be lower. However, I try to avoid these questions and break things up into smaller parts when necessary, but looking at the data it seems it still happens.

2. What will you do differently as a result of what you learned?

I'm not sure I'm going to change anything. As I said, I think about what I do every time I give an exam and teach. Teaching is an evolving process that I go through every day of a semester, and this data has not really changed what I think of teaching linear algebra.

III.11.3. Formative Assessment. I use quiz results to gauge student understanding. I quiz on material within one week of the students' first exposure to topic. Based on those results, I may review an idea that was not grasped.

When students attend my office hours I will ask them how they are thinking about a problem, or how their understanding has changed from when they came in for help and when they leave.
III.12. Math 129, Basic Linear Algebra, fourth section

III.12.1. Final exam.

Question 1.
Alexis has a pocket full of change. She has a total of 16 quarters, dimes, and nickels whose total value is three dollars. She has twice as many dimes as she does nickels. Set up a system of linear equations to determine how much of each denomination she has. Solve the system using whatever method you wish.

Outcomes assessed: Systems of linear equations
Degree of success: A 97% B 0% C 3% D 0% E 0%

Question 2.
Consider the infinite dimensional vector space consisting of all power series in the variable $x$, denoted $\mathbb{P}_\infty = \{ \sum_0^\infty a_n x^n \mid a_n \text{ is real} \}$ with the basis $B_\infty = \{ 1, x, x^2, x^3, \ldots \}$. Let $D : \mathbb{P}_\infty \to \mathbb{P}_\infty$ by acting on the basis elements with the rule $D(x^r) = r x^{r-1}$. And consider $\iota : \mathbb{P}_\infty \to \mathbb{P}_\infty$ by acting on the basis elements with the rule $\iota(x^r) = \frac{1}{r+1} x^{r+1}$. These are two linear transformations. $D$ is the derivative and $\iota$ is the antiderivative in calculus.

a.) Find the matrix for $D$ relative to $B_\infty$.
b.) Find the matrix for $\iota$ relative to $B_\infty$.
c.) What is the dimension of the null space of $D$? 
d.) What is the dimension of the null space of $\iota$?
e.) What is $D(\iota(x^r))$? 
f.) What is $\iota(D(x^r))$? 
g.) Are the matrices you found in parts a and b inverses of one another? 
h.) Are the matrices you found in parts a and b row equivalent?

Outcomes assessed: Inverses, subspaces, row operations
Degree of success: A 46% B 18% C 27% D 3% E 6%

Question 3.
Consider $F = \{ f \mid f$ is a real valued function$\}$. Define the sum of two functions by $(f + g)(x) = f(x) + g(x)$ and a scalar times a function as $(cf)(x) = cf(x)$.

a.) Under these definitions is $F$ a vector space? 
b.) Is the set $T = \{ 1, \cos x, \sin x, \sin^2 x, \cos^2 x, \sin^3 x, \cos^3 x, \ldots \}$ a linearly independent set? 
c.) Does there exist a linearly independent set with infinitely many elements from $F$? If so give an example of one, if not explain why not. 
d.) If $F$ is a vector space, does it have a basis?
Outcomes assessed: Subspaces, linear independence
Degree of success: A 24%  B 46%  C 21%  D 9%  E %

**Question 4.**

Let $A = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$.

a.) Find the characteristic polynomial for $A$.

b.) Show that the characteristic polynomial in part b annihilates $A$.

c.) What are the eigenvalues associated to $A$?

d.) Describe the eigenspaces in terms of a basis for each eigenvalue you found in part c.

e.) Write an equation that relates all eigenvectors to their associated eigenvalue.

Outcomes assessed: Eigenvalues, eigenspaces, dimension
Degree of success: A 79%  B 3%  C 18%  D 0%  E 0%

**Question 5.**

Consider the following system:

\[
\begin{align*}
3x_1 + 4x_2 - 5x_3 + x_4 &= 0 \\
2x_1 + 4x_2 + x_3 &= 0 \\
5x_3 - x_4 &= 0 \\
x_1 - x_2 + x_3 &= 0
\end{align*}
\]

a.) Write down the associated matrix for this homogenous system, call it $A$.

b.) Write down the row-reduced echelon matrix for $A$, call it $E_A$

c.) Write the solution set in parametric vector form.

d.) How does this compare to the Nul $A$?

e.) Find a basis for Col $A$.

f.) What are the dimensions of Nul $A$ and Col $A$?

Outcomes assessed: Systems of linear equations, echelon form, subspaces, homogeneous systems, rank, dimension
Degree of success: A 91%  B 0%  C 6%  D 3%  E 0%

**Question 6.**

An $n \times n$ matrix is called skew-symmetric if $A^T = -A$.

a.) Write down a $3 \times 3$ matrix that is skew-symmetric.
b.) Calculate the determinant of the matrix you wrote in part a.
c.) Prove that if \( n \) is odd and \( A \) is a skew-symmetric matrix, then \( \det A = 0 \).
d.) Is the statement in c true if \( n \) is even?

Outcomes assessed: Determinants
Degree of success: A 61%  B 0%  C 27%  D 12%  E 0%

III.12.2. Reflection.
1. What did you learn from these data?
I learned in a very concrete way all the topics covered in the final I wrote, as well as how
the students understood individual concepts.
2. What will you do differently as a result of what you learned?
In the future, I think it might be useful to write down all topics each question on an exam
covers, to ensure that all topics get the appropriate amount of coverage.

III.12.3. Formative Assessment. I give several quizzes throughout the semester. I use
them to gauge how well students are learning the material I am teaching them. If I find
quiz scores are below what I would like them, I usually readdress some topics.
I also use my office hours to gauge how students are learning, and to pick up on any
"holes" I might need to fill. I often ask students how the class is going for them, and
what their likes and dislikes are. I also try to pay attention to the "attitude" of the class.
If they are not as talkative as normal, I often slow down until they start asking questions
or can more easily answer the questions I propose in class. I propose a lot of questions in
class, and pay attention to who is what. If I do not get a broad range of people answering
questions I might single out a section of the room, to make sure all students are learning.
III.13. Math 129, Basic Linear Algebra, fifth section

III.13.1. Final exam.

Question 1.

Let \( A = \begin{bmatrix} 2 & 3 & -3 \\ -4 & 1 & 2k \\ 6 & 2 & -1 \end{bmatrix} \) and \( b = \begin{bmatrix} 3m \\ 2m \\ 4 \end{bmatrix} \). Find all values of \( k \) and \( m \) such that \( Ax = b \) has:

a.) a unique solution
b.) no solution
c.) infinitely many solutions

Outcomes assessed: Systems of linear equations
Degree of success: A 28% B 26% C 14% D 18% E 14%

Question 2.

Let \( A = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \), \( B = \begin{bmatrix} -3 & 5 \\ 1 & 1 \end{bmatrix} \) and \( C = \begin{bmatrix} 2 & 2 \\ 3 & 2 \end{bmatrix} \). Find \((AB)^{-1}C^{-1}\).

Outcomes assessed: Matrix operations
Degree of success: A 54% B 7% C 14% D 18% E 7%

Question 3.

a.) Define a transformation \( T : \mathbb{M}_{2\times2} \to \mathbb{R} \) by \( T(A) = \det(A) \). Determine if \( T \) is a linear transformation. Clearly justify your conclusion.

b.) Let \( H = \left\{ \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \mid a = -2, f = 2e + d \right\} \). Is \( H \) a subspace of \( \mathbb{M}_{2\times2} \)?

Outcomes assessed: Subspaces
Degree of success: A 18% B 4% C 7% D 43% E 28%

Question 4.

Consider the matrix \( A = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix} \).

a.) Find a basis for Row \( A \).
b.) Find a basis for Nul \( A \).
c.) Find a basis for Col \( A \).
d.) Find rank \( A \).
e.) Find $\dim(\text{Nul } A)$.

Outcomes assessed: Rank, dimension
Degree of success: A 11%  B 29%  C 7%  D 21%  E 32%

**Question 5.**

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ 3 & 2 & -2 \end{bmatrix}$.

a.) Find the eigenvalues of matrix $A$.

b.) Find the bases for the eigenspaces associated with the eigenvalues of matrix $A$.

Outcomes assessed: Eigenvalues
Degree of success: A 7%  B 25%  C 21%  D 14%  E 33%

**Question 6.**

Define a transformation $T : \mathbb{M}_{2 \times 2} \rightarrow \mathbb{R}$ by $T \left( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = a + b - c - d$. Determine the dimension of the kernel of $T$ by finding the basis.

Outcomes assessed: Dimension
Degree of success: A 3%  B 7%  C 4%  D 4%  E 82%

**Question 7.**

Let $H = \{ a + bx^2 + cx^4 | a, b, c \in \mathbb{R} \}$. Calculate the dimension of $H$ in $\mathbb{P}_4$ by finding a basis for $H$. Justify that your basis works.

Outcomes assessed: Dimension
Degree of success: A 7%  B 7%  C 7%  D 15%  E 64%

**Question 8.**

Let $A$ be a $7 \times 12$ matrix.

a.) Is it possible that $\dim(\text{Nul}(A)) = 4$? Explain why or why not.

b.) Is it possible that $\dim(\text{Nul}(A)) = 8$? Explain why or why not.

c.) If $\dim(\text{Nul}(A)) = 5$, then does some subset of the columns of $A$ form a basis for $\mathbb{R}^7$? Explain why or why not.

Outcomes assessed: Rank, dimension
Degree of success: A 25%  B 0%  C 7%  D 11%  E 57%
Question 9.
Let $V, W$ and $Z$ be vector spaces. Assume $L : V \rightarrow W$ and $T : W \rightarrow Z$ are linear transformations. Show that the composition function $T \circ L : V \rightarrow Z$ given by $(T \circ L)(x) = T(L(x))$ is a linear transformation.

Outcomes assessed: Vectors in $n$-space

Degree of success: A 18%  B 0%  C 0%  D 4%  E 78%

Question 10.
Let $A = \begin{bmatrix} 4 & 2 & 2 & 0 \\ 2 & 5 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Calculate $\det A$.

Outcomes assessed: Determinants

Degree of success: A 86%  B 11%  C 0%  D 0%  E 3%

Question 11. (Bonus)
An $n \times n$ matrix $C$ is called symmetric if $C = C^T$. Let $W$ be an $n \times 1$ matrix such that $W^T W = 1$. Then $n \times n$ matrix

$$H = I_n - 2WW^T$$

is called a **Householder matrix**. Show that $H$ is symmetric.

Outcomes assessed: Matrix operations

Degree of success: A 0%  B 0%  C 3%  D 4%  E 93%

III.13.2. **Reflection.**

1. What did you learn from these data?

I notice, the students are struggling with some particular topics in linear algebra such as Vector spaces, dimension of the kernel, bases, matrix operations, linearly independence.

2. What will you do differently as a result of what you learned?

If I teach linear algebra again I probably will spend more time covering these particular topics, show them more examples.
III.14. Math 129, Basic Linear Algebra, sixth section

III.14.1. Final exam.

Question 1.
Determine if the following statements are true or false. Justify your response.

a.) If \( \mathbf{u}, \mathbf{v} \) and \( \mathbf{w} \) are solutions to \( A \mathbf{x} = \mathbf{b} \), then \( 2 \mathbf{u} - 3 \mathbf{v} + 2 \mathbf{w} \) is also a solution.

b.) If \( A \mathbf{x} = \mathbf{b} \) has a unique solution, then \( A \) has an equal number of rows and columns.

Outcomes assessed: Systems of linear equations, vectors in \( n \)-space

Degree of success: A 40%  B 14%  C 23%  D 23%  E 0%

Question 2.

Let \( B = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & k \\ 1 & 1 & k^2 \end{bmatrix} \). Find all values of \( k \) such that the columns of \( B \) are linearly independent.

Outcomes assessed: Row operations, homogeneous systems, linear independence

Degree of success: A 80%  B 10%  C 10%  D 0%  E 0%

Question 3.

Let \( A = \begin{bmatrix} x + 2 & 2 & -4 & 6 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & x + 3 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2x & -5 \\ 0 & 1 & 2 \end{bmatrix} \).

Find all values of \( x \) such that \( \det A = \det B \).

Outcomes assessed: Determinants

Degree of success: A 20%  B 10%  C 27%  D 43%  E 0%

Question 4.

Let \( C = \begin{bmatrix} 2 & -1 & 1 & -1 \\ 2 & 0 & -1 & 4 \\ -4 & 3 & -4 & 8 \\ 6 & 0 & -3 & 12 \end{bmatrix} \).

a.) Find a basis for Row \( C \).

b.) What is the \( \dim(\text{Row}(C)) \)?

c.) What is the \( \dim(\text{Nul}(C)) \)?
d.) What is the \( \dim(\text{Col}(C)) \)?

Outcomes assessed: Row operations, echelon form, linear independence, dimension
Degree of success: A 54%  B 33%  C 3%  D 10%  E 0%

**Question 5.**

Let \( D = \begin{bmatrix} 2 & 2 & 5 \\ 4 & 5 & 12 \\ -2 & -3 & -6 \end{bmatrix} \). 

a.) Find two bases for \( \text{Col} \, D \).

b.) Find \( \text{rank} \, D \).

Outcomes assessed: Row operations, echelon form, vectors in \( n \)-space, linear independence, rank
Degree of success: A 17%  B 17%  C 13%  D 40%  E 13%

**Question 6.**

Let \( A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 2 & 2 & 0 \end{bmatrix} \).

a.) Find the eigenvalues of the matrix \( A \).

b.) Find a basis for the eigenspace associated one of the eigenvalues from part a. (You may choose which eigenvalue you use).

Outcomes assessed: Eigenvalues, eigenvectors
Degree of success: A 37%  B 27%  C 20%  D 13%  E 3%

**Question 7.**

Let \( T : \mathbb{R}^3 \to \mathbb{R}^2 \) by \( T \left( \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) = \begin{bmatrix} 2a - b \\ b \end{bmatrix} \).

a.) Find a basis for \( \text{Ker} \, T \).

b.) What is the \( \dim(\text{Ker} \, T) \)?

Outcomes assessed: Subspaces, dimension
Degree of success: A 10%  B 13%  C 20%  D 54%  E 3%

**Question 8.**

Let \( V = \text{Span} \left\{ \begin{bmatrix} 2 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ -6 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 5 \\ 3 \end{bmatrix} \right\} \). Find a basis for \( V \).
Outcomes assessed: Systems of linear equations, subspaces, linear independence  
Degree of success: A 3% B 64% C 3% D 30% E 0%

Question 9.
Let $A$ be a $n \times n$ invertible matrix. Determine if the following statements are true sometimes, always, or never. You do not need to justify your response.

a.) $A\vec{e}_1 = \vec{0}$

b.) $\text{rank}(A) = n$

c.) The columns of $A$ are basis for $\mathbb{R}^n$

d.) The rows of $A$ are basis for $\mathbb{R}^n$

e.) $\det(A) = n$

Outcomes assessed: Determinants, linear independence, rank  
Degree of success: A 40% B 13% C 27% D 17% E 3%

Question 10. (Bonus)
In class I gave the vector space of continuous functions as an example of an infinite dimensional vector space. Show, or explain why, this vector space is infinite dimensional.

Outcomes assessed:
Degree of success: A 0% B 0% C 0% D 30% E 70%

III.14.2. Reflection.

1. What did you learn from these data?

The data regarding the exams is both interesting but not surprising. It is not surprising because I think about the problems I put on exams beforehand. Generally, based on student performance on quizzes and conversations with students during office hours I know how the exam is going to go. There are questions that I put on every exam I write (as Ive taught this course multiple times) that repeatedly give students trouble. Even when I give multiple examples and explanations of an idea, some problems still give students difficulty. I dont think the problem is that these problems are too challenging, as I try to keep most of the exam in line with problems that are given in the text.

The data is interesting because it confirms some things I have thought, but being confronted with it, makes me rethink what I do instructionally. For instance, certain problems have distributions with large numbers of students grouped in two categories, with few students in the category between the categories. I wonder if the flaw is in the question, the grading, or if there is no flaw. Many questions do tend to be contingent on one central
idea. If a student does not understand that idea, the score will obviously be lower. However, I try to avoid these questions and break things up into smaller parts when necessary, but looking at the data it seems it still happens.

2. What will you do differently as a result of what you learned?

I'm not sure I'm going to change anything. As I said, I think about what I do every time I give an exam and teach. Teaching is an evolving process that I go through every day of a semester, and this data has not really changed what I think of teaching linear algebra.

III.14.3. **Formative Assessment.** I use quiz results to gauge student understanding. I quiz on material within one week of the students’ first exposure to topic. Based on those results, I may review an idea that was not grasped.

When students attend my office hours I will ask them how they are thinking about a problem, or how their understanding has changed from when they came in for help and when they leave.
III.15.1. **Final exam.**

**Question 1.**

Let \( A = \begin{bmatrix} 12 & 20 & 26 & 16 \\ -6 & -10 & -13 & -8 \\ 3 & 6 & 6 & 5 \end{bmatrix} \).

a.) Write the solutions to \( A \vec{x} = \vec{0} \) in parametric vector form.

b.) Suppose \( A \begin{bmatrix} 0 \\ 2 \\ -3 \\ 11 \end{bmatrix} = \vec{b} \). Describe all solutions to \( A \vec{x} = \vec{b} \) in parametric vector form.

Outcomes assessed: Systems of linear equations, row operations, echelon form, homogeneous

Degree of success: A 15% B 27% C 19% D 39% E 0%

**Question 2.**

Let \( A \) and \( B \) be two matrices, where \( A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \) and \( AB = \begin{bmatrix} -4 & 2 & -2 \\ -17 & 7 & -13 \end{bmatrix} \). Find \( B \).

Outcomes assessed: Matrix operations

Degree of success: A 77% B 15% C 4% D 4% E 0%

**Question 3.**

a.) Let \( A \) be an \( m \times n \) matrix. Show, without quoting a theorem, that \( \text{Nul} \ A \) is a subspace of \( \mathbb{R}^n \).

b.) Show that \( H = \{ a + bx + cx^3 \mid a, b, c \in \mathbb{R} \} \) is a subspace of \( \mathbb{P}_3 \).

Outcomes assessed: Subspaces

Degree of success: A 65% B 8% C 27% D 0% E 0%

**Question 4.**

Let \( A = \begin{bmatrix} -2 & 4 & -6 & 6 & -14 \\ 3 & -6 & 9 & -5 & 13 \\ 1 & -2 & 3 & -2 & 5 \end{bmatrix} \).

a.) Find a basis for the \( \text{Col} \ A \)
b.) What is the dim(Row(A))?  
c.) What is the dim(Nul(A))?  
d.) What is the dim(Col(A))?  

Outcomes assessed: Row operations, echelon form, linear independence, dimension

Degree of success: A 62% B 15% C 12% D 8% E 3%

Question 5.

Let \( A = \begin{bmatrix} -1 & 4 \\ 3 & 6 \end{bmatrix} \), \( B = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} \) and \( C = \begin{bmatrix} 12 & 7 \\ -1 & k \end{bmatrix} \). Find all values of \( k \) such that \( \{A, B, C\} \) is a linearly independent set in \( M_{2 \times 2} \).

Outcomes assessed: Homogeneous systems, linear independence

Degree of success: A 50% B 4% C 31% D 15% E 0%

Question 6.

Let \( B = \begin{bmatrix} -1 & 0 & 1 \\ -3 & 0 & 6 \\ -4 & 0 & 3 \end{bmatrix} \).

a.) Find the eigenvalues of matrix \( B \).

b.) Find a basis for the eigenspace associated to one of the eigenvalues from part a. (You may choose which eigenvalue you use).

Outcomes assessed: Eigenvalues, eigenvectors

Degree of success: A 23% B 11% C 35% D 31% E 0%

Question 7.

Let \( V = \text{Span}\{x^3 + x^2 + x + 3, x^3 + x, 2x^2 + 6, x^2 - 3x + 1\} \). Find a basis for \( V \). Justify why your answer is a basis.

Outcomes assessed: Subspaces, linear independence

Degree of success: A 19% B 0% C 12% D 65% E 4%

Question 8.

Determine if the following statements are true or false. Justify your response.

a.) If \( V \) is a vector space with \( \text{dim}(V) = 78 \), then there is a subspace \( H \) of \( V \) such that \( \text{dim}(H) = 20 \).

b.) Any linear independent set of vectors in a vector space \( V \) can be expanded to a basis for \( V \).

Outcomes assessed: Subspaces, linear independence
Question 9.
Let $A$ be a $7 \times 11$ matrix. Justify your responses

a.) Is it possible $\dim(\text{Nul}(A)) = 3$?
b.) Is it possible $\dim(\text{Nul}(A)) = 6$?
c.) Suppose $\text{Row}(A) = \mathbb{R}^7$. What is $\text{rank}(A)$?

Outcomes assessed: Subspaces, homogeneous systems, dimension, rank

Question 10. (Bonus)
Let $V$ be the vector space of all polynomials. Find $\dim(V)$.

Outcomes assessed: Dimension

III.15.2. Reflection.

1. What did you learn from these data?

The data regarding the exams is both interesting but not surprising. It is not surprising because I think about the problems I put on exams beforehand. Generally, based on student performance on quizzes and conversations with students during office hours I know how the exam is going to go. There are questions that I put on every exam I write (as I’ve taught this course multiple times) that repeatedly give students trouble. Even when I give multiple examples and explanations of an idea, some problems still give students difficulty. I don’t think the problem is that these problems are too challenging, as I try to keep most of the exam in line with problems that are given in the text.

The data is interesting because it confirms some things I have thought, but being confronted with it, makes me rethink what I do instructionally. For instance, certain problems have distributions with large numbers of students grouped in two categories, with few students in the category between the categories. I wonder if the flaw is in the question, the grading, or if there is no flaw. Many questions do tend to be contingent on one central idea. If a student does not understand that idea, the score will obviously be lower. However, I try to avoid these questions and break things up into smaller parts when necessary, but looking at the data it seems it still happens.

2. What will you do differently as a result of what you learned?

I’m not sure I’m going to change anything. As I said, I think about what I do every time I give an exam and teach. Teaching is an evolving process that I go through every day of a semester, and this data has not really changed what I think of teaching linear algebra.
III.15.3. **Formative Assessment.** I use quiz results to gauge student understanding. I quiz on material within one week of the students’ first exposure to topic. Based on those results, I may review an idea that was not grasped.

When students attend my office hours I will ask them how they are thinking about a problem, or how their understanding has changed from when they came in for help and when they leave.
III.16. Math 129, Basic Linear Algebra, eighth section

III.16.1. Final exam.

Question 1.

Find a general solution and a nontrivial solution to the following linear system:

\[
\begin{align*}
5w + 3x + 2y + z &= 0 \\
2w + 5x + 6y - z &= 0 \\
4w + 8x - 12y + 3z &= 0
\end{align*}
\]

Outcomes assessed: Systems of linear equations, homogeneous systems

Degree of success: A 45%  B 8%  C 26%  D 18%  E 3%

Question 2.

Let \( A = \begin{bmatrix} 2 & 8 & 3 \\ 7 & 4 & 9 \\ 5 & 4 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \).

a.) Compute \( \det(A) \) without the use of a calculator.

b.) Find a basis for \( \text{Col}(A) \) without the use of a calculator.

c.) Explain why any 2 rows of \( B \) would form a basis for \( \text{Row}(B) \).

d.) Compute \( \dim(\text{Nul}(B)) \).

Outcomes assessed: Row operations, determinants, rank, dimension

Degree of success: A 14%  B 33%  C 35%  D 12%  E 6%

Question 3.

Let \( A \) be an \( n \times n \) matrix such that \( A \) and \( A^T \) are not row equivalent. Is it possible that \( A \) is row equivalent to

\[
\begin{bmatrix}
1 & 8 & 7 \\
0 & \sin x & -\cos x \\
0 & \cos x & \sin x
\end{bmatrix}
\]

Outcomes assessed: Determinants

Degree of success: A 21%  B 6%  C 16%  D 10%  E 47%

Question 4.

Give an example of two \( 3 \times 3 \) matrices (call them \( A \) and \( B \)) where \( \det A = 3 = \dim(\text{Row}(A)) \) but \( A \) and \( B \) are not row equivalent.

Outcomes assessed: Determinants, dimension

Degree of success: A 41%  B 1%  C 3%  D 45%  E 10%
Question 5.

Let \( S = \text{Span}\left\{ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}, \begin{bmatrix} 9 & 10 \\ 11 & 12 \end{bmatrix} \right\} \).

a.) Show that \( S \) spans \( M_{2 \times 2} \) or explain why this is not possible.
b.) Does \( M_{2 \times 2} \) have a basis containing every vector in \( S \)?

Outcomes assessed: Subspaces, linear independence
Degree of success: A 20%   B 3%   C 30%   D 19%   E 28%

Question 6.

Define \( L : \mathbb{P}_2 \rightarrow M_{2 \times 2} \) by \( L(a + bx + cx^2) = \begin{bmatrix} a + 2b + 3c & 4a + 5b + 6c \\ 7a + 8b + 9c & a + b + c \end{bmatrix} \).

a.) Show that \( L \) is a linear transformation.
b.) Show that \( \text{Ker}(L) \) is a subspace of \( \mathbb{P}_2 \).
c.) Let \( f, g, h, i \in \mathbb{P}_2 \). Without finding a basis for \( \text{Ker}(L) \), explain why \( \{f, g, h, i\} \) cannot possibly be a basis for \( \text{Ker}(L) \).
d.) Compute \( \text{dim}(\text{Ker}(L)) \).

Outcomes assessed: Subspaces, dimension, linear independence
Degree of success: A 4%   B 5%   C 18%   D 45%   E 28%

Question 7.

Let \( f, g, h \in \mathbb{P}_2 \), where \( f \neq 0 \) and \( \deg f < \deg g < \deg h \). If \( k \in \mathbb{P}_2 \), then is it necessarily true that \( k \in \text{Span}\{f, g, h\} \)?

Outcomes assessed: Linear independence
Degree of success: A 8%   B 12%   C 13%   D 29%   E 38%

III.16.2. Reflection.

1. What did you learn from these data?

The students have a terribly difficult time grasping the concepts of linear combinations, particularly with respect to the notion of spanning. This doesn’t surprise me given that the prerequisite for this course is only Math 104.

2. What will you do differently as a result of what you learned?

I continue to stress computations early on in the course so as to enhance their intuition as to what is really going on. Also, I continue to try to improve my exam-writing skills so the students can more accurately demonstrate what they have and have not learned.
III.16.3. **Formative Assessment.** Discuss wording of problems with students. Take end of semester eval’s very seriously. Not a big fan of small group work - too often too few do the work for too many. In class I present different solutions to the same problem and try to get some feedback as to which solution the students prefer. I try to get a sense of whether or not any exam questions came out of left field - I want the student who has studied and is prepared to have a good sense as to what should be coming at them.
III.17. Math 146, Applied Calculus I, first section

III.17.1. Final exam.

Question 1.
Find the derivative of the function
\[ f(x) = \frac{2x^4 + 3x}{8x}. \]

Hint: Before calculating the derivative, simplify the function using properties of the natural logarithmic function \( \ln \).

Outcomes assessed: Logarithmic functions, derivatives
Degree of success: A 8%  B 1%  C 5%  D 5%  E 81%

Question 2.
Calculate the derivative of the function
\[ g(t) = \frac{e^{3t} + e^{-3t}}{e^{3t}}. \]

Simplify your resulting derivative.

Outcomes assessed: Exponential functions, derivatives
Degree of success: A 2%  B 0%  C 0%  D 2%  E 96%

Question 3.
Find the derivative of the function
\[ h(x) = e^{\ln x}. \]

Outcomes assessed: Exponential functions, derivatives
Degree of success: A 2%  B 6%  C 0%  D 0%  E 92%

Question 4.
Find the inflection points of the function
\[ f(x) = \ln(x^2 + 1)^3. \]

Hint: An inflection point of a function \( f(x) \) is a point \( a \) satisfying \( f''(a) = 0 \).

Outcomes assessed: Derivatives
Degree of success: A 0%  B 5%  C 0%  D 1%  E 94%
Question 5.
Find the horizontal asymptote of the function
\[ f(x) = \frac{3x^2}{1 + x^2}. \]
Hint: For horizontal asymptotes find the limits (if they exist) \( \lim_{x \to \infty} f(x) \) and \( \lim_{x \to -\infty} f(x) \).
Outcomes assessed: Limits
Degree of success: A 14%  B 1%  C 2%  D 3%  E 80%

Question 6.
One wishes to build a rectangular enclosure for his rabbit against the side of his house. He has bought 240 feet of fencing. What are the dimensions (width and length) of the largest area that he can enclose?
Outcomes assessed: Optimization
Degree of success: A 27%  B 3%  C 1%  D 6%  E 63%

Question 7.
Evaluate the definite integral
\[ \int_{-1}^{1} \frac{x}{x + 2} \, dx. \]
Hint: Use the method of integration by substitution.
Outcomes assessed: Integrals, techniques of integration
Degree of success: A 0%  B 6%  C 2%  D 3%  E 89%

Question 8.
Evaluate the definite integral
\[ \int_{0}^{4} \sqrt{5+2x} \, dx. \]
Hint: Use the method of integration by substitution.
Outcomes assessed: Integrals, techniques of integration
Degree of success: A 1%  B 3%  C 5%  D 2%  E 89%

Question 9.
Find the area \( A \) of the region enclosed by the graph of the function
\[ f(x) = 1 - x^2 \]
and the \( x \)-axis, for \( 0 \leq x \leq 1 \). Hint: Employ the FTC.
Outcomes assessed: Integrals
Question 10.
Calculate the derivative of the function
\[ f(x) = \frac{(x^2 + 1)^3}{(x^3 + 1)^4(3 + 2x^7)^7} \]
by using the method of logarithmic differentiation.

Outcomes assessed: Derivatives
Degree of success: A 19% B 6% C 3% D 6% E 66%

III.17.2. Reflection.
1. What did you learn from these data?
The data you collected is very interesting. I have no doubt you have invested lot of efforts in obtaining the data.

2. What will you do differently as a result of what you learned?
I have no intention to modify my teaching methods, which are designed to teach calculus in classes of students who are not familiar with basic high school algebraic operations.

III.17.3. Formative Assessment. Instructor administers an unofficial student evaluation form each semester.
III.18. Math 146, Applied Calculus I, second section

III.18.1. **Final exam.**

**Question 1.**
Choose the phrase that best describes "derivative".

- instantaneous rate of change
- slope of a secant line
- Power Rule
- velocity

Outcomes assessed: Derivatives, rate of change

Degree of success: A 89%  B 0%  C 0%  D 0%  E 11%

**Question 2.**
Choose the phrase that best describes "definite integral".

- area under a graph
- limiting value of Riemann sums
- antiderivative
- distance

Outcomes assessed: Integral

Degree of success: A 62%  B 0%  C 0%  D 0%  E 38%

**Question 3.**
Researchers in Spain have proposed that the model $W(t) = 650.1(1 - e^{-0.038t})^3$ can be used to estimate the weight of Retinta beef cows of various ages. Here, $W(t)$ is the weight (in kg) of a $t$-month-old cow.

a.) Interpret the statements $W(48) \approx 383.42$ and $W'(48) \approx 8.41$. Include appropriate units.

b.) Show a complete sequence of steps that verifies that $W'(48) \approx 8.41$.

Outcomes assessed: Derivatives, rates of change, applications

Degree of success: A 45%  B 5%  C 39%  D 10%  E 1%

**Question 4.**
Consider the following graph of a function $f(x)$. Assume that gridlines each mark one unit.
Arrange the following quantities in increasing order:

- $f'(-6)$
- $f''(1)$
- $f(-1)$
- $\int_{-3}^{0} f(x) \, dx$
- the average rate of change of $f(x)$ on the interval $[-4, -1]$

Outcomes assessed: Derivatives, rates of change, integral

Degree of success: A 38%  B 0%  C 53%  D 9%  E 0%

**Question 5.**

Below is the graph of $y = f'(t)$ for some function $f(t)$. Assume that the gridlines each mark one unit.

Use the graph to answer the questions that follow.

a.) Estimate all intervals where $f(t)$ is increasing.

b.) Is it true that $f$ has a relative maximum value when $t \approx 1.5$? Explain your claim.
c.) Estimate all intervals where the graph of \( y = f(t) \) is concave up. Express your result in interval notation.

Outcomes assessed: Derivatives, optimization, applications
Degree of success: A 19%  B 11%  C 40%  D 24%  E 6%

Question 6.
The velocity of a projectile is described by \( v(t) = 0.01t^3 - 0.4t^2 + 4t \) for \( 0 \leq t \leq 20 \), where \( t \) is measured in seconds and \( v(t) \) is measured in centimeters per second.

a.) What does the quantity \( \frac{v(15) - v(10)}{15 - 10} \) represent? What is the appropriate label for this quantity?

b.) Is it true that the acceleration of the projectile at \( t = 20 \) seconds is 0 centimeters per second per second? Justify your claim.

c.) Calculate the average velocity of the projectile over the indicated 20-second period of time. Label appropriately.

Outcomes assessed: Derivatives, integral, applications
Degree of success: A 4%  B 1%  C 43%  D 49%  E 3%

Question 7.
A container of water is being heated in such a way that the temperature of the water is currently increasing at a rate of 4°F per minute, and that rate of change is itself increasing at a continuous rate of 15% each successive minute. Given this, find how much the water temperature will increase over the course of the next 10 minutes, accurate to two decimal places.

Outcomes assessed: Exponential functions, integral, applications
Degree of success: A 14%  B 3%  C 8%  D 66%  E 9%

Question 8.
A marginal cost function and a marginal revenue function are given below. The fixed costs are $250.
a.) Estimate the total cost to produce 50 units.

b.) Calculate the revenue from the 31st through the 60th units produced and sold.

c.) Let \( x \) represent number of units. Find a formula for the revenue function, \( R(x) \).

Outcomes assessed: Integral, applications

Degree of success: A 11%  B 6%  C 42%  D 35%  E 6%

Question 9. (Bonus)

A closed box with a square base is to have a volume of 24 cubic feet. The material for the sides, top, and base costs $0.25, $0.50, and $1 per square foot, respectively.

a.) Let \( x \) represent the length, in feet, of one of the edges of the base of the box. Write a function \( C(x) \) that gives the total cost of the box in terms of \( x \).

b.) Find the cost of the least expensive box. You must show clearly how you have used your function from above and justify your conclusion.

Outcomes assessed: Optimization

Degree of success: A 21%  B 5%  C 26%  D 25%  E 23%

III.18.2. Reflection.

1. What did you learn from these data?

Students continue to struggle with information described graphically and recalling some basic ideas from earlier work (exponential functions, for instance). Problems that involve basic technique are handled relatively well.

2. What will you do differently as a result of what you learned?

I would attempt to integrate ”old” ideas throughout the semester and make sure that, when the topic is appropriate, that students complete some instructor-reviewed problems that approach a topic from non-algebraic perspectives.
III.19. Math 147, Applied Calculus II, first section

III.19.1. Final exam.

Question 1.
Suppose $4000 is deposited into an account earning an interest rate of 2.8%, with interest compounded weekly. How long will it take for the amount of money in the account to double?

Outcomes assessed: Applications
Degree of success: A 65%    B 0%    C 20%    D 15%    E 0%

Question 2.
Monty just signed a new contract for an annual salary of $53,000. He is guaranteed a 3% raise each year he decides to stay on the job.
   a.) If he completes five years of service, what will his salary be for the sixth year on the job?
   b.) If Monty works at this job for a total of 30 years and gets a 3% raise each year, how much money will he earn over the entire 30 years?

Outcomes assessed: Applications, geometric series
Degree of success: A 39%    B 4%    C 0%    D 55%    E 2%

Question 3.
Karen wants to start putting away a fixed amount of money each month for three years to save up for a bathroom remodeling project. She knows she can put the money into an account that earns interest at an annual rate of 4% compounded monthly. She estimates that she will need $8,000 for the project.
   a.) How much money should she deposit into the account each month to achieve her goal?
   b.) If Kathy can afford to put away $200 each month, what annual interest rate, compounded monthly, does her money need to earn in order to reach her goal of $8,000 in three years? Give your response as a percentage, correct to two decimal places.

Outcomes assessed: Applications, geometric series
Degree of success: A 37%    B 8%    C 14%    D 41%    E 0%

Question 4.
Which periodic rate has a higher APY: a weekly rate of 0.2865%, or a monthly rate of 1.23%? Be sure to clearly justify your conclusion.
Question 5.

A couple can make quarterly deposits of $450 into an account that earns an annual interest rate of 5% compounded quarterly. In five years, when one of their children leaves for college, they will cease making payments into the account. Starting one quarter after that last deposit, equal quarterly payments will be issued from the account for four years to help pay college expenses. After the final payment, the account should be empty.

(a) Calculate the quarterly payments that will be issued.
(b) How much interest does the money in the account ultimately earn?

Question 6.

A detective finds a murder victim at 7:00 am. The temperature of the body is measured at 89.8°F. One hour later, the temperature of the body is 88°F. The temperature of the room has been maintained at a constant 72°F.

(a) Assuming that the temperature, $y$, of the body obeys Newton's Law of Cooling, write a differential equation for $y$, and give all appropriate initial conditions.
(b) Estimate the time at which the murder occurred. Assume a body temperature of 98.6°F at the time of the murder.

Question 7.

Suppose that you borrow $120,000 to purchase a home. The loan will be repaid over the course of 30 years with equal monthly payments. The annual interest rate on the loan is 5.75%, compounded monthly, giving monthly payments of $700.29.

(a) How much interest is due on the first payment? How much interest is due on the second payment?
(b) What will the unpaid loan balance be after 10 years of payments?
(c) If you can afford monthly payments of $850, how much could you have borrowed under the same loan terms (interest rate and number of payments)?
Question 8. (Bonus)
A superball is dropped from a height of 120 feet. The ball bounces up and down several times, and on each bounce the ball rises to $\frac{4}{5}$ of the height of the previous bounce.

a.) Set up a series (sum) that shows the total distance (both up and down) the ball has traveled at the moment it hits the ground for the 3rd time. You don’t actually need to calculate the distance.

b.) How many times must the ball have hit the ground if it has traveled a total distance (both up and down) of 1020 feet? Clearly justify your conclusion.

Outcomes assessed: Applications, geometric series
Degree of success: A 2%  B 0%  C 22%  D 29%  E 47%

III.19.2. Reflection.
1. What did you learn from these data?
Students tend to handle the final exam topics very well, probably because the number of topics is kept relatively limited compared to the vast array of specialized applications that they have worked on throughout the semester. There continues to be weakness when dealing with a problem that becomes algebraically challenging, in that many students don’t seem to have an alternate strategy for finding the desired information.

2. What will you do differently as a result of what you learned?
I have developed several different problems dealing with various topics that require students to employ a method of solution that is not simply algorithmic application of a formula.
III.20. Math 147, Applied Calculus II, second section

III.20.1. Final exam.

Question 1.
Suppose $4000 is deposited into an account earning an interest rate of 2.8%, with interest compounded weekly. How long will it take for the amount of money in the account to double?

Outcomes assessed: Applications
Degree of success: A 69%  B 0%  C 12%  D 10%  E 9%

Question 2.
Monty just signed a new contract for an annual salary of $53,000. He is guaranteed a 3% raise each year he decides to stay on the job.

a.) If he completes five years of service, what will his salary be for the sixth year on the job?

b.) If Monty works at this job for a total of 30 years and gets a 3% raise each year, how much money will he earn over the entire 30 years?

Outcomes assessed: Applications, geometric series
Degree of success: A 57%  B 1%  C 29%  D 13%  E 0%

Question 3.
Karen wants to start putting away a fixed amount of money each month for three years to save up for a bathroom remodeling project. She knows she can put the money into an account that earns interest at an annual rate of 4% compounded monthly. She estimates that she will need $8,000 for the project.

a.) How much money should she deposit into the account each month to achieve her goal?

b.) If Kathy can afford to put away $200 each month, what annual interest rate, compounded monthly, does her money need to earn in order to reach her goal of $8,000 in three years? Give your response as a percentage, correct to two decimal places.

Outcomes assessed: Applications, geometric series
Degree of success: A 56%  B 9%  C 26%  D 7%  E 2%

Question 4.
A 500-gallon holding tank contains 100 gallons of water. Initially, each gallon of water in the tank contains 2 pounds of pollutants. Water containing 6 pounds of pollutants
per gallon enters the tank at a rate of 50 gallons per hour, and the uniformly mixed water is released from the tank at a rate of 30 gallons per hour. Let \( P(t) \) denote the number of pounds of pollutants in the tank after \( t \) hours. State a differential equation and initial condition that could be used to explicitly find \( P(t) \). Do not attempt to solve the differential equation.

Outcomes assessed: Applications

Degree of success: A 47%  B 8%  C 26%  D 16%  E 3%

**Question 5.**

A couple can make quarterly deposits of $450 into an account that earns an annual interest rate of 5% compounded quarterly. In five years, when one of their children leaves for college, they will cease making payments into the account. Starting one quarter after that last deposit, equal quarterly payments will be issued from the account for four years to help pay college expenses. After the final payment, the account should be empty.

a.) Calculate the quarterly payments that will be issued.

b.) How much interest does the money in the account ultimately earn?

Outcomes assessed: Applications, geometric series

Degree of success: A 53%  B 3%  C 33%  D 8%  E 3%

**Question 6.**

A detective finds a murder victim at 7:00 am. The temperature of the body is measured at 89.8\(^{\circ}\)F. One hour later, the temperature of the body is 88\(^{\circ}\)F. The temperature of the room has been maintained at a constant 72\(^{\circ}\)F.

a.) Assuming that the temperature, \( y \), of the body obeys Newtons Law of Cooling, write a differential equation for \( y \), and give all appropriate initial conditions.

b.) Estimate the time at which the murder occurred. Assume a body temperature of 98.6\(^{\circ}\)F at the time of the murder.

Outcomes assessed: Introduction to differential equations, applications

Degree of success: A 38%  B 29%  C 20%  D 9%  E 4%

**Question 7.**

Suppose that you borrow $120,000 to purchase a home. The loan will be repaid over the course of 30 years with equal monthly payments. The annual interest rate on the loan is 5.75\%, compounded monthly, giving monthly payments of $700.29.

a.) How much interest is due on the first payment? How much interest is due on the second payment?

b.) What will the unpaid loan balance be after 10 years of payments?
c.) If you can afford monthly payments of $850, how much could you have borrowed under the same loan terms (interest rate and number of payments)?

Outcomes assessed: Applications, geometric series
Degree of success: A 38%  B 4%  C 44%  D 10%  E 4%

**Question 8. (Bonus)**

A superball is dropped from a height of 120 feet. The ball bounces up and down several times, and on each bounce the ball rises to 4/5 of the height of the previous bounce.

a.) Set up a series (sum) that shows the total distance (both up and down) the ball has traveled at the moment it hits the ground for the 3rd time. You don’t actually need to calculate the distance.

b.) How many times must the ball have hit the ground if it has traveled a total distance (both up and down) of 1020 feet? Clearly justify your conclusion.

Outcomes assessed: Applications, geometric series
Degree of success: A 10%  B 1%  C 32%  D 46%  E 11%

**III.20.2. Reflection.**

1. What did you learn from these data?
2. What will you do differently as a result of what you learned?
III.21. Math 165, University Calculus I, first section

III.21.1. Final exam.

Question 1.
In each case, find $\frac{dy}{dx}$. Fully justify your answers.

a.) $y = \tan(x^3)e^{cosx}$

b.) $\frac{x}{1+xy} = x + y^2$

c.) $y = \frac{x^{4x}}{(1+x)^3e^x}$

Outcomes assessed: Differentiation

Degree of success: A 7%    B 27%    C 29%    D 26%    E 11%

Question 2.
Find the following integrals. Justify your answers.

a.) $\int 4\sqrt{x} - \frac{2}{1+x^2} dx$

b.) $\int \frac{x^2}{1+x^3} dx$

c.) $\int_1^2 4x + \frac{2}{x} dx$

d.) $\int_1^4 \frac{1}{2+\sqrt{t}} dt$

Outcomes assessed: Integration, Fundamental Theorem of Calculus

Degree of success: A 1%    B 5%    C 36%    D 38%    E 20%

Question 3.
Find the following limits. Justify your answers.

a.) $\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$

b.) $\lim_{x \to \infty} \frac{(2x^2 + 2)^4}{x^8 + 7}$

Outcomes assessed: Limits

Degree of success: A 6%    B 6%    C 14%    D 41%    E 33%

Question 4.
The graph of $y = f(x)$ is shown below.
a.) Put the following $x$-values in order of increasing values of $f'(x)$. $x = -1, \frac{1}{2}, 3$.

b.) What are the critical values of $f(x)$?

c.) At which of the following values is $f''(x)$ negative? $x = -1, \frac{1}{2}, 2$.

d.) Is $\int_{-1}^{1} f(x)dx$ positive or negative?

Outcomes assessed: Continuity, differentiation, integration

Degree of success: A 45%  B 0%  C 30%  D 18%  E 7%

Question 5.

45th street runs north-south. 52nd avenue runs east-west. Alan is driving north along 45th street at 50 km/h. Ben is driving east along 52nd avenue. At exactly 3pm, Alan is 3 km away and Ben is 4 km away from the junction of 45th and 52nd and the distance between them is growing by 60 km/h. How fast is Ben driving at 3pm?

Outcomes assessed: Differentiation, applications

Degree of success: A 45%  B 12%  C 3%  D 9%  E 31%

Question 6.

A particle has initial velocity 4 m/s and acceleration $a(t) = -2t$ m/s$^2$.

a.) Find the velocity of the particle as a function of time.

b.) Find the net change in position of the particle over the interval $0 \leq t \leq 3$.

c.) Find the total distance travelled over the interval $0 \leq t \leq 3$.

Outcomes assessed: Integration, applications

Degree of success: A 17%  B 23%  C 18%  D 31%  E 11%

Question 7.

A cardboard box is shaped to be a cuboid with a square base. Find the maximum possible volume if the total surface area is 24 ft$^2$. Make sure to justify that your answer is the maximum.
Outcomes assessed: Differentiation, applications
Degree of success: A 14% B 12% C 4% D 20% E 50%

**Question 8.**

The age of a bacteria population in days is calculated by the formula \( A = 3 \ln P \) where \( P \) is the size of the population.

a.) If the population is measured to be \( P = e^{10} \) with a possible error of up to 3000 bacteria. Use linear approximation to estimate the maximum error in the calculated age.

b.) Again the population is measured to be \( P = e^{10} \). This time there is a percentage error of up to 2% in the measurement. Estimate the maximum percentage error in the calculated age.

Outcomes assessed: Differentiation, applications
Degree of success: A 2% B 4% C 0% D 21% E 73%

III.21.2. **Reflection.**

1. What did you learn from these data?

The data didn’t really tell me anything I didn’t already know but confirmed a few things. The homeworks and the midterms had already split the C/D students from the A/B students. The final was intended to give a clear split between A and B but did this a little too well with the average scores being lower than expected. While I don’t feel any question was too difficult, the number of questions with more difficult parts was probably too high.

2. What will you do differently as a result of what you learned?

In the future, a smaller proportion of the exam will be aimed at the top portion of the class.
III.22. Math 165, University Calculus I, second section

III.22.1. Final exam.

Question 1.

Evaluate the following limits if they exist.

a.) \( \lim_{x \to -2} \frac{x^4 - 4x - 24}{x^3 + x^2 - x + 2} \)

b.) \( \lim_{x \to \infty} x \sin\left(\frac{1}{x}\right) \)

c.) \( \lim_{x \to 0} \left( \frac{1}{ax + 1} \right)^{\frac{1}{x^2}} \)

d.) \( \lim_{x \to \infty} \frac{x + \sin(x)}{x\sqrt{x}} \)

e.) \( \lim_{x \to \infty} \frac{2x^3}{\sqrt{x^6 + 1}} \)

Outcomes assessed: Limits, differentiation

Degree of success: A 6%  B 29%  C 26%  D 23%  E 16%

Question 2.

Find the derivatives of the following functions.

a.) \( f(x) = g(\sqrt{x})h(x^2) \)

b.) \( f(x) = x^{(ax)^{(bx)}} \)

c.) \( h(x) = \tan^{-1}(\sin(x^2 + 1)) \)

d.) \( k(x) = \int_{e^{2x}}^{x^3} \cos(t^2)dt \)

Outcomes assessed: Differentiation, Fundamental Theorem of Calculus

Degree of success: A 10%  B 30%  C 25%  D 19%  E 16%

Question 3.

Evaluate the following integrals.

a.) \( \int_{-1}^{1} \frac{\tan^{-1}(x) \tan^2(x^2)}{x^3 + 1} dx \)

b.) \( \int \frac{\sin(2x)}{\sin^2(x) + a^2} dx, \ a \neq 0 \)

c.) \( \int_{\ln(2)}^{\infty} \frac{e^x}{2e^x + 4} dx \)
Question 4.
Use the definition of the derivative to find the derivative of \( f(x) = \sqrt{x^2 + 4x} \).

Question 5.
Use the definition of the definite integral to find \( \int_{0}^{4} (4x - 2) \, dx \).

Question 6.
Find the area of the largest triangle that can be inscribed inside a circle of radius \( R \).

Question 7.
A conical tank (with circular base on top) is being filled with water at a constant rate. The tank has base radius 6 feet and height 8 feet. If the water level is rising at 1 inch per second when the water is 4 feet deep, at what rate is the tank being filled (in cubic feet per second)?

Question 8.
Sketch the graph of the curve 
\( f(x) = \ln |x^{\frac{1}{3}} + 1| \)
For your convenience, the first two derivatives are given by 
\[ f'(x) = \frac{1}{3x^{\frac{2}{3}}(x^{\frac{1}{3}} + 1)} \]
and 
\[ f''(x) = \frac{-(3x^{\frac{1}{3}} + 2)}{9x^{\frac{2}{3}}(x^{\frac{1}{3}} + 1)^2}. \]
Question 9.
A curve is defined implicitly by the equation $y^3 - x^3 = xy + 1$. Find the tangent line to this curve at the point $(0, 1)$.
Outcomes assessed: Differentiation, applications
Degree of success: A 30%  B 23%  C 11%  D 9%  E 27%

Question 10.
A spherical asteroid has a diameter of 100 miles ($\pm \frac{1}{2}$ miles). Use a linear approximation to estimate the maximum error in using the 100 mile measurement to compute the surface area of the asteroid. What is the relative error? (The surface area of a sphere of radius $R$ is given by $S = 4\pi R^2$.)
Outcomes assessed: Differentiation, applications
Degree of success: A 6%  B 7%  C 23%  D 12%  E 52%

III.22.2. Reflection.
1. What did you learn from these data?
Given the fact that the computations used in some of the problems were not for the faint of heart, I was pleased with the outcomes of some of the results (limits for the most part and derivatives). I am a bit concerned with the integration, but I have noticed that students struggle with that since it is the last section before the final (and they are not tested on it before this). I was also concerned with the weaknesses on applied problems (especially since they were easier than the ones on the midterms).

2. What will you do differently as a result of what you learned?
For the future, I am thinking about how to get them to be able to use/apply the calculus more effectively. Historically, multiple-step conceptual problems (even with easy steps) are difficult for the students. I will inform the committee when I have conquered this problem. I would also like to give another assessment on integration, but do not know if this is practical since time is of the essence at that point in the semester.

III.22.3. Formative Assessment. I had each TA do an "early on" survey to make sure that the mechanics of their teaching was OK (or needed to be adjusted). I conduct extensive review sessions that I use to give a barometer on where the students are in their development. This has prompted me to give more sections/concepts extra attention. Additionally, after every exam, I conduct informal interviews of students to see where they are in the course and what their weaknesses and strengths are. This has also helped me to steer the course in the (I hope) appropriate direction.
I also keep a very complete spreadsheet on the course. I use analysis of the data over the past years (I have taught this course on sequence for a decade) to help assess student progress.
III.23. Math 165, University Calculus I, third section

III.23.1. Final exam.

Question 1.

Evaluate the following limits.

a.) \( \lim_{x \to 1} \frac{x^2 - 1}{x^2 - x} \)

Outcomes assessed: Limits, continuity
Degree of success: A 99% B 0% C 0% D 0% E 1%

b.) \( \lim_{z \to 16} \frac{4 - \sqrt{z}}{z - 16} \)

Outcomes assessed: Limits, continuity
Degree of success: A 66% B 24% C 8% D 1% E 1%

c.) \( \lim_{x \to 0} \left( \frac{1}{3x + 1} \right)^{\frac{1}{x}} \)

Outcomes assessed: Limits, continuity
Degree of success: A 22% B 24% C 13% D 29% E 12%

d.) \( \lim_{x \to \infty} \frac{x^4 + 5x}{2x^4 + 1} \)

Outcomes assessed: Limits, continuity
Degree of success: A 94% B 2% C 0% D 3% E 1%

e.) \( \lim_{x \to 0} x^{12} \cos \left( \frac{1}{x^3} \right) \)

Outcomes assessed: Limits, continuity
Degree of success: A 54% B 5% C 23% D 3% E 15%

Question 2.

A rectangular poster is to have a total area of 180 square inches with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions of the poster will give the largest printed area?
Outcomes assessed: Differentiation, applications
Degree of success: A 50%  B 14%  C 16%  D 20%  E 0%

**Question 3.**
Find the vertical and horizontal asymptotes of the function
\[ f(x) = \frac{\sqrt{x^2 + 8}}{4 - 9x}. \]

Outcomes assessed: Limits, continuity
Degree of success: A 29%  B 2%  C 64%  D 2%  E 3%

**Question 4.**
Find an equation of the tangent line to the curve at the point \((5, \frac{9}{4})\). Note that \(x\) is your independent variable.
\[ \frac{x^2}{16} - \frac{y^2}{9} = 1 \]

Outcomes assessed: Differentiation, applications
Degree of success: A 38%  B 27%  C 12%  D 17%  E 6%

**Question 5.**
How are limits and derivatives related to one another?
Outcomes assessed: Limits, differentiation
Degree of success: A 72%  B 0%  C 23%  D 0%  E 5%

**Question 6.**
Find the derivatives \(\frac{dy}{dx}\) for parts a) through c) and find \(B'(x)\) for part d).

a.) \( y = \tan^{-1}(8x)(5x + 9)^4 + 732 \)

Outcomes assessed: Differentiation
Degree of success: A 45%  B 44%  C 8%  D 1%  E 2%

b.) \( y = \frac{\ln(x^2 + \cos(2x))}{\sqrt{\tan x}} \)

Outcomes assessed: Differentiation
Degree of success: A 45%  B 26%  C 19%  D 8%  E 2%

c.) \( y = (2x)^{e^x} \)
Outcomes assessed: Differentiation
Degree of success: A 29%  B 17%  C 14%  D 24%  E 16%

d.) $B(x) = \int_x^9 \sin \sqrt{3} dt$

Outcomes assessed: Differentiation, Fundamental Theorem of Calculus
Degree of success: A 42%  B 1%  C 29%  D 10%  E 18%

Question 7.
Consider $f(x) = 8x^2 - 7x + 2000679$. On what interval(s) is the function concave down?
Outcomes assessed: Differentiation, applications
Degree of success: A 81%  B 7%  C 3%  D 9%  E 0%

Question 8.
What is the Mean Value Theorem used for? Provide enough information, so that I know you understand when the theorem is useful and what conditions have to hold in order to be able to use it.
Outcomes assessed: Mean Value Theorem
Degree of success: A 44%  B 14%  C 28%  D 8%  E 6%

Question 9.
Evaluate the following integrals.

a.) $\int e^x \sin(e^x) dx$

Outcomes assessed: Integration
Degree of success: A 45%  B 28%  C 11%  D 10%  E 6%

b.) $\int \frac{1}{x \ln x} dx$

Outcomes assessed: Integration
Degree of success: A 45%  B 13%  C 19%  D 17%  E 6%

c.) $\int_0^1 (x^2 + 1)^2 dx$
Outcomes assessed: Integration, Fundamental Theorem of Calculus
Degree of success: A 48%  B 10%  C 11%  D 27%  E 4%

d.) \[ \int_{799.2}^{799.2} \tan \left( \frac{\sqrt{\sin 5x + x + 2}}{x^2} \right) \, dx \]

Outcomes assessed: Integration, Fundamental Theorem of Calculus
Degree of success: A 97%  B 0%  C 0%  D 0%  E 3%

III.23.2. Reflection.

1. What did you learn from these data?

I learned that my students are doing very well with many topics such as derivatives, integration, Fundamental Theorem of Calculus. However, I was surprised that they still need work in some areas such as finding the vertical and horizontal asymptotes of functions and limits problems such as 1c.

2. What will you do differently as a result of what you learned?

The breakdown of the exam outcomes is very helpful in terms of guiding my future teaching. I will critically examine (and in turn modify) how I teach the areas/topics where my students had the most difficulties on this final exam to help prepare the next group of students better in those areas.

III.23.3. Formative Assessment. After each exam, I ask the large lecture class to respond to questions related to the material. They include, but are not limited to, the following questions.

1. How do you think the test went?
2. What could you do to do better on the next exam?
3. What can I do to help you do better on the next exam?
4. What topics do you think you understood the best/least on the material that was tested?

I use this information to inform what topics I need to review and help the students learn. We revisit these topics of interest during the first five minutes of class each day in the form of start-up problems. I also use their feedback to modify my instruction to help meet their learning needs.

I’m still not sure how to answer the quantitative reasoning question. Here’s an attempt though.

I write my exams so that students have to go beyond memorizing a series of steps to solve a problem. They must reason though the question to follow the logical way to complete
it. I give a variety of types of questions on exams so that students have to develop both conceptual and procedural knowledge as well as critical thinking skills. During class, I have the students reason through problems on their own without me telling them the answer. Then when they are stuck, I help them reason through the way to solve the problem.
III.24. Math 165, University Calculus I, fourth section

III.24.1. Final exam.

Question 1.
Evaluate the following integrals.

a.) \[ \int x \sin(x^2 + 1) \, dx \]

b.) \[ \int_1^4 \sqrt{x}(1 + x) \, dx \]

c.) \[ \int_0^1 x^2(1 + 2x^3)^5 \, dx \]

d.) \[ \int \frac{(\ln x)^2}{x} \, dx \]

Outcomes assessed: Integration, Fundamental Theorem of Calculus
Degree of success: A 4% B 35% C 31% D 29% E 1%

Question 2.
Find the derivatives of the following functions

a.) \[ f(x) = x^3 \cos x \]

b.) \[ g(x) = \frac{x^3 + 2x + 1}{\ln(x^2 + 1)} \]

c.) \[ h(x) = \sin(e^{x^2}) \]

Outcomes assessed: Differentiation
Degree of success: A 37% B 37% C 21% D 5% E 0%

Question 3.
If a snowball melts so that its surface area decreases at a rate of 1 cm\(^2\)/min, find the rate at which the diameter decreases when the diameter is 10 cm. (Recall that the surface area \( S \) of a sphere of radius \( R \) is given by \( S = 4\pi R^2 \)).

Outcomes assessed: Differentiation, applications
Degree of success: A 11% B 12% C 40% D 28% E 9%

Question 4.
Let \( f(x) = \frac{x^2}{x^2 - 1} \).

a.) Find the horizontal asymptotes.
b.) Find the vertical asymptotes.
c.) Find the intervals on which \( f \) is increasing or decreasing.
d.) Find the intervals of concavity and the inflection points.
e.) Find the local maximum and minimum values of \( f \).
f.) Sketch the graph of \( f \).

Outcomes assessed: Applications
Degree of success: A 8%  B 20%  C 45%  D 26%  E 1%

**Question 5.**
Find the point on the parabola \( y = 2x^2 \) that is closest to the point \((1, 4)\).

Outcomes assessed: Applications
Degree of success: A 10%  B 3%  C 3%  D 13%  E 71%

**Question 6.**
Use linear approximation to estimate \( \sqrt{99.8} \).

Outcomes assessed: Applications
Degree of success: A 23%  B 33%  C 12%  D 17%  E 15%

**Question 7.**
Find the absolute maximum and minimum values of the function \( f(x) = \frac{x}{x^2 + 1} \) on the interval \([0, 2]\).

Outcomes assessed: Applications
Degree of success: A 34%  B 19%  C 9%  D 34%  E 4%

**Question 8.**
Use implicit differentiation to find an equation of the tangent line to the curve
\[ x^2 + 2xy - y^2 + x = 2 \]
at the point \((1, 2)\).

Outcomes assessed: Differentiation
Degree of success: A 27%  B 26%  C 14%  D 28%  E 5%

**Question 9.**
Evaluate the limits.

a.) \( \lim_{x \to 1} \frac{\ln x}{x - 1} \)
b.) \( \lim_{x \to \infty} \sqrt{x^2 + x} - x \)
c.) \( \lim_{x \to 0^+} x^x \)

Outcomes assessed: Limits

Degree of success: A 5%  B 13%  C 25%  D 52%  E 5%

**Question 10.**

Show that the equation \( \cos x = x^3 \) has at least one real root.

Outcomes assessed: Continuity

Degree of success: A 4%  B 14%  C 25%  D 50%  E 7%

**III.24.2. Reflection.**

1. What did you learn from these data?

In most part it confirmed my suspicions that I had after finishing grading the test. Most students have the basic skills of differentiation and integration but struggle when they have to use them in applications. In the same vein, the students do well when dealing with “classic type” problems whose solutions follow well-established procedures, but not so well when facing problems that require thinking and explaining a mathematical argument. Perhaps not surprising, the concept of limit and its correct (i.e. mathematical) interpretation is not well understood by most students.

2. What will you do differently as a result of what you learned?

I will insist more on the concept of limit and its applications. I will continue to consider problems that involve “non-algorithmic” thinking and insist more on learning how to apply the calculus tools in applications.
III.25. Math 165, University Calculus I, fifth section

III.25.1. Final exam.

Question 1.

Find all values of $a$ so that the function $f(x) = \begin{cases} x + 1 & \text{if } x \leq a \\ 3 - x & \text{if } x > a \end{cases}$ is continuous on $\mathbb{R}$.

Outcomes assessed: Limits, continuity

Degree of success: A 7%  B 2%  C 0%  D 78%  E 13%

Question 2.

Evaluate the limit $\lim_{x \to \infty} e^{x-x^2}$.

Outcomes assessed: Limits

Degree of success: A 14%  B 10%  C 23%  D 7%  E 46%

Question 3.

Calculate $\frac{dy}{dx}$ if $y \cos(x^2) = x \cos(y^2)$.

Outcomes assessed: Differentiation

Degree of success: A 42%  B 18%  C 16%  D 9%  E 15%

Question 4.

Evaluate the integral $\int \cot x \ln(\sin x) dx$.

Outcomes assessed: Integration

Degree of success: A 15%  B 22%  C 23%  D 11%  E 29%

Question 5.

Evaluate the limit $\lim_{x \to \infty} x^4 e^{-2x}$.

Outcomes assessed: Limits, applications

Degree of success: A 42%  B 0%  C 30%  D 23%  E 5%
Question 6.
Evaluate the integral, if it exists
\[ \int_{\pi/4}^{\pi/4} \frac{x^4 \tan x}{2 + \cos x} \, dx. \]

Outcomes assessed: Integration, applications
Degree of success: A 36%  B 0%  C 3%  D 47%  E 14%

Question 7.
Find the derivative of the function
\[ f(x) = \int_0^{x^2} \sin(t^2) \, dt. \]

Outcomes assessed: Differentiation, Fundamental Theorem of Calculus
Degree of success: A 29%  B 2%  C 2%  D 2%  E 65%

III.25.2. Reflection.
1. What did you learn from these data?
I did not learn anything new. Beyond the statistics that I already had, I do not see what conclusions can be drawn.

2. What will you do differently as a result of what you learned?
I cannot decide now what I will do differently next time I will be teaching the class. It will all depend on how the students will interact in classroom.

III.25.3. Formative Assessment. During the semester, when doing examples I guide the students to find the answers themselves, I usually ask them to vote on an exercise answer that seems “obvious” but on a careful analysis is not the right one, and on numerical examples I ask half of the students to use approximations early in the solving of an exercise, the other half to use approximations later and then compare and interpret the results.
Part IV. Client Department/College Reflection

Kevin McCaul, Dean of College of Science and Mathematics

I thought the most interesting part were the instructors’ comments. Unfortunately, much of the time the instructors either found the assessments irrelevant to their teaching or didn’t care that many students performed poorly (i.e., they didn’t plan to do anything different)!

Eakalak Khan, Ph.D., P.E., Chair and Professor, Civil Engineering Dept.

Below is my feedback on the data collected for your course assessments. My feedback is applied strictly to MATH 129 and 165 because our civil engineering students normally do not take the other courses covered in the report.

1. Based on the data presented in the report, I feel the courses are serving our students well. Course outcomes are clear in term of what students are expected to be able to do after completing the courses. In general, final examination problems were appropriate and relevant tools to measure the outcomes.

2. I would like to see the instructors critically analyze the assessment data to find out why students do not perform well on certain outcomes or topics. Here are some good examples. One of the MATH 129 instructors suggests that MATH 104 may not be adequate as a prerequisite. One of the MATH 165 instructors comments that poor performances on integration could be because the topic was covered at the end of semester and students were not tested on the topic before the final examination. Without knowing the causes of the weak performances, it would be difficult to improve student learning. I am surprised that some of the instructors did not learn anything from the assessment data. These instructors should come up with assessment methods that would provide them with useful data for improving student learning.

3. I have a concern on how some of the instructors intend to design and use examinations to split grades. This suggests that these instructors operate on an assumption that some students would not perform well in their courses no matter what. I believe that instructors should never give up on student learning. I also have some comments on the assessment. The assessment method is very systematic and well designed in general. I commend your assessment committee on this effort. Has your committee thought of using more than one assessment tool (final examination)? For example, some instructors in my department use quizzes along with examinations as the assessment tools and have found out that students tend
to perform better on quizzes and likely because they are usually given not long after the topics are lectured and students still retain the knowledge. I observe substantial variations in assessment data among different sections of the same course. For example, students tend to perform better in section 4 of MATH 129 and section 3 of MATH 165. Has your committee looked into the causes of these variations? Maybe a summary of the data based on learning outcomes among different sections would be a good start to examine these variations. I also notice that some sections include bonus questions in the examination. However, the performances on these bonus questions are exceptionally poor to the point that their value and purpose are questionable.
Part V. Mathematics Department Reflection

Response 1.
Although a few instructors said that they did not learn anything from the assessment and do not plan to change anything, my impression was not as negative as the reflections collected in Part IV of the report. I see that most instructors are concerned about student learning, and they are suggesting many specific ways to improve it, such as spending more time on certain topics, asking the questions in a different manner, having students work in groups, using extra assessment tools, etc. The implementation of these new ways can be a challenge, given the time constraints that we always have. But, on the whole, my impression of the value of this report and the reflections it has prompted is quite positive. As for the instructors that could not find ways to improve student learning, I think they could benefit from meeting with instructors teaching the same sections, who can share their ideas with them.

Response 2.
I find the degree of success for the first section of Math 146 to be inexcusably low. It is disappointing, to say the least, that the instructor blames the students for the low scores in his/her remarks and writes that he/she will not make any changes in the course.

On the other hand, I was glad to see many thoughtful comments from other instructors. It is good to see that many instructors in the department are taking the department’s assessment activities seriously.

Response 3.
As a member of the Department of Mathematics, I am concerned that we as a department be sure that students learn. This is especially true in the beginning (100-level) courses, since these courses are often the gateways to further work in mathematics or other major fields. I think that concern is well addressed by considering final exams.

Improvement of assessment, I feel, may be attained by assessing the students' retention at the beginning of courses to which the assessed courses are prerequisite.
Part VI. Summary

The Mathematics Department at NDSU is working to assess how its students are meeting the course objectives currently outlined in the Bulletin. To this end, final examinations were analyzed question-by-question for each section of certain courses. Then the instructor for each course examined the data. Responses were collected from instructors, mathematics faculty members, and client departments.

It was found that some instructors learned a lot from the data and will teach and/or review material for their exams differently in the future. Some instructors, however, did not find these data very helpful.

The Mathematics Assessment Committee will continue to improve its plan to assess student learning.