

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

https://www.ndsu.edu/math/outreach/nd_talent_search/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by January 31, 2024, using the form on the website. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the **second set** of problems:

1. Is the number $2023^4 + 2^{2026} \cdot 121^{2024}$ prime? Justify your answer.
2. Let x be a complex number. Find all the **real** values of k for which

$$x + \frac{1}{x} = x^7 + \frac{1}{x^7} = k.$$

Explain your reasoning.

3. Three different buses stop at my bus stop: Line A, line B and line C buses. The total trip length of a line A bus is 40 minutes, for line B one hour, and for line C 25 minutes. Today at 7am all three buses were at my stop. What time will they all coincide there again?
4. We intercept a coded message that has been encrypted as follows:
 - The numbers $0, \dots, 9$ are coded as themselves.
 - The letters A, \dots, Z are assigned the numbers $A = 10, B = 11, \dots, Z = 35$.
 - Blank space is encoded as 36.

Each number is then encrypted by multiplying it by a fixed (but unknown) number α and adding another unknown number β . If the resulting number is larger than 36, it is then written modulo 36 and transformed back to the original alphabet.

The message we have intercepted is OH7F86BB46R3627O266BB9. There are no zeros, only the vowel O. We know that the last three letters represent the signature, 007. What does the message say, and what are α, β ?

5. How many 6-letter passwords can we create with the characters A,B,C,D,E,1,2,3,4,5 if no two letters can be adjacent, no two numbers can be adjacent, and no character is used more than once?