

## Talent Search: Round 3 (Final Round)

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

<http://math.ndsu.nodak.edu/talent/2013/>

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

**It is not necessary to have participated in previous rounds. Every student is encouraged to participate in this round.**

The students who submit a significant number of mathematically sound solutions for each of the rounds will be rewarded with various prizes, including a one-year subscription to a science/mathematics magazine of their choice. The best participants who decide to attend NDSU and major in the mathematical sciences will also be rewarded with scholarships.

**Please submit your solutions by email to [maria.alfonseca@ndsu.edu](mailto:maria.alfonseca@ndsu.edu), by February 28, 2014.** Alternatively, solutions may be sent by regular mail to:

Talent Search  
c/o Maria Alfonseca  
Mathematics NDSU Dept.# 2750  
PO BOX 6050  
Fargo, ND 58108-6050

**Please do not forget to include your name, postal address, school, and e-mail address.**

Here is the third and last set of problems:

1. Find  $\frac{z}{y+z}$ , given that  $\frac{z}{x+y} = a$  and  $\frac{y}{x+z} = b$ .
2. We write all the irreducible fractions  $p/q$ , where  $p, q$  are positive and  $q < 100$  in ascending order of value (for example,  $4/9 = 0.444\dots$  comes after  $2/5 = 0.4$ ). Between which two fractions does  $5/8$  stand?
3. The police department of a city is investigating a burglary. There are four suspects, A, B, C, and D. The police have established that the following facts are true:
  - Fact 1: If both A and B are guilty, then C was their accomplice.
  - Fact 2: If A is guilty, then either B or C were accomplices.
  - Fact 3: If C is guilty, then D was his accomplice.
  - Fact 4: If A is innocent, then D is guilty.

From these facts it can be proven that one of the four suspects is necessarily guilty. Who?

(Note: An accomplice is considered guilty).

4. Let ABCD be a rhombus (vertices are named consecutively, hence the two vertices closer to A are B and D). Consider all the points M on the plane of the rhombus, verifying that the sum of the angles AMB and CMD equals 180 degrees. Describe how such points are located.
5. Prove:  $4ab \leq (a + b)^2 \leq 2(a + b)^2$ . Under what circumstances does equality hold?
6. A family of four (father, mother, son, and daughter) are hiking in the evening and reach a shaky bridge when it is already dark. The bridge can hold no more than two people at a time, and those crossing need to have the lantern with them. There is just one lantern for the whole family. Suppose that it takes the son 1 minute to cross the bridge, the daughter 3 minutes, the father 8 minutes and the mother 10 minutes. Can the entire family cross the bridge in 20 minutes? If so, how? (When any two persons cross the bridge, their speed is equal to that of the slower one).