Talent Search

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

http://math.ndsu.nodak.edu/talent/2013/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes, including a one-year subscription to a science/mathematics magazine of their choice. The best participants who decide to attend NDSU and major in the mathematical sciences will also be rewarded with scholarships.

Please submit your solutions by email to maria.alfonseca@ndsu.edu, by October 18, 2013. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the first set of problems:

1. Find all the pairs of natural numbers x and y such that both $x^2 + 3y$ and $y^2 + 3x$ are squares of whole numbers.

2. One hundred students participated in a mathematics competition consisting of four problems. No student solved every problem. The first problem was solved by exactly 90 students; the second by exactly 80, the third by exactly 70, and the fourth by exactly 60 students. A prize was awarded only to each student who solved both the third and fourth problems. How many prizes were awarded?

3. We are given a circle with center O and a straight line that intersects this circle at two points A and B, in such a way that the chord AB is not a diameter of the circle.

Describe the location of the midpoints of all the chords of the circle such that their endpoints lie on different sides of the given line.

Hint: Note that for any point K within the circle (with K different than the center O), there is only one chord such that K is its midpoint.

4. How many natural numbers less than 10^8 are there, with sum of digits equal to 5?

5. A fair coin is flipped until a head occurs. What is the probability that a head first appears on an even numbered toss?

6. What is the greatest possible number of points of intersection for eight distinct lines on a plane?