Talent Search: Round 2

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

http://math.ndsu.nodak.edu/talent/2013/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

It is not necessary to have participated in the first round. Every student is encouraged to participate in this round.

The students who submit a significant number of mathematically sound solutions for each of the rounds will be rewarded with various prizes, including a one-year subscription to a science/mathematics magazine of their choice. The best participants who decide to attend NDSU and major in the mathematical sciences will also be rewarded with scholarships.

Please submit your solutions by email to maria.alfonseca@ndsu.edu, by December 17, 2013. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the second set of problems:

1. Find all real solutions to the equation $(x^2 + 100)^2 = (x^3 - 100)^3$.

2. A farmer takes 10 tons of watermelons to the market. When he leaves, the water content of the watermelons is 99% of their weight. Along the way the watermelons dry out, so that when the farmer arrives to the market their water content is 98% of their weight. What is the weight of the load of watermelons when the farmer arrives to the market?

3. A bag contains 150 black marbles and 75 white marbles. A person draws two marbles from the bag. If the drawn marbles are one black and one white, the person replaces the white marble in the bag and discards the black marble. If the two drawn marbles are both the same color, they are discarded and one black marble is placed in the bag (there is an unlimited supply of black marbles). The process is repeated. Eventually there will be just one marble left in the bag (why?). What is its color?

(Turn over for problems 4 and 5)

4. Prove that $2222^{5555} + 5555^{2222}$ is divisible by 7.

5. To a person standing on the surface of Saturn, from what latitude will Saturn's rings appear to be widest?

   Explanation (see attached picture): We assume that Saturn is perfectly spherical with a radius of 60,000 km, and that the rings are perfectly circular. The distance from the center of Saturn to the inner ring is 67,000 km, and the distance from the center of Saturn to the outer ring is 140,000 km. The latitude is the angle φ above the equator, and the rings will appear widest when the angle α is biggest.