Talent Search

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

https://www.ndsu.edu/math/ongoing_events/nd_talent_search/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by October 23, 2105, using the form on the website. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the first set of problems:

1. We pile oranges forming a pyramid with a triangular base. A one-layer pyramid will have only one orange, a two-layer pyramid will have 4 oranges (3 in the bottom layer and 1 on the top), a three-layer pyramid will have 10 oranges (6 in the bottom layer, 3 in the middle and 1 on top).

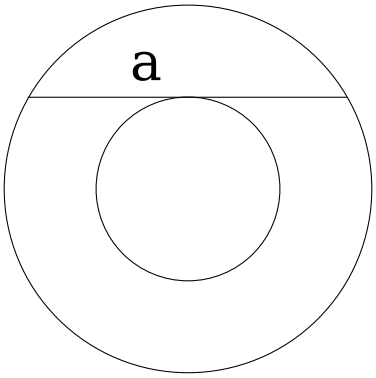
   (a) Find the general formula for the total number of oranges in a triangular pyramid with n layers.

   (b) Find the general formula for the total number of oranges in a square pyramid with n layers.

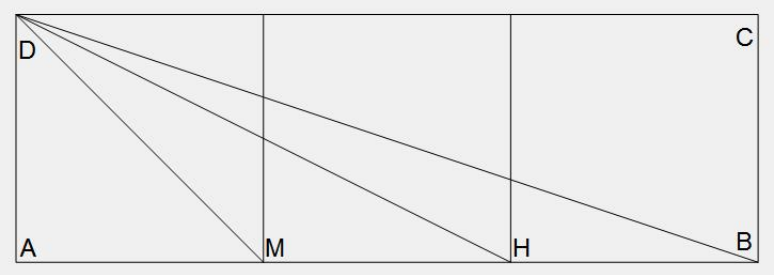
   (c) Given a number N, is it possible to place N oranges both as a triangular pyramid and as a square pyramid? (the pyramids do not necessarily need to have the same number of layers, just the same number of total oranges).

2. On the graph of $y = x^2$, how many points with integer coordinates are there that determine a line of slope 1?

3. We have two concentric circles, and we know that the length of a chord tangent to the inner circle is a. Find the area of the annulus between the two circles as a formula depending on a (you have all the information you need for this problem).



4. Three congruent squares with bases AM, MH and HB are put next to each other to form a rectangle $ABCD$. Show, using a geometric argument, that $\angle AMD + \angle AHD + \angle ABD = 90^o$.



5. Bellini and Cellini were two Rennaisance Italian goldsmiths who specialized in making jewelry boxes with an inscription on them. Every box made by Bellini had a true inscription, every box made by Cellini had a false inscription. At the Rennaisance Fair we run across a stand with three Bellini or Cellini jewelry boxes, with a sign saying that only one of them contains a jewel, and the person who can correctly determine which one, and explain why, will win the jewel. These are the 3 inscriptions:

   Golden box: If the jewel is in the silver box, then the silver box was made by Bellini.
   Silver box: If the jewel is in this box, then the golden box was made by Cellini.
   Lead box: The box that contains the jewel was made by Cellini.

   Where is the jewel and who made each box?