Talent Search

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

https://www.ndsu.edu/math/ongoing_events/nd_talent_search/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by January 29, 2016, using the form on the website. Alternatively, solutions may be sent by email to maria.alfonseca@ndsu.edu, or by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the second set of problems:

1. Find the value of the sum

$$\frac{1}{2}\cdot 1+\frac{1}{4}\cdot 2+\frac{1}{8}\cdot 3+\frac{1}{16}\cdot 4+\cdots\frac{1}{2^n}\cdot n+\cdots .$$

   You can use properties of infinite series, or a geometric method. Do not use derivatives.

2. For each of the platonic solids (tetrahedron, cube, octahedron, dodecahedron, icosahedron), find a path along the edges that passes through each vertex exactly once, and finishes at the same vertex where it started.

3. Which of the following events is more likely? (Explain your answer)

   (a) The appearance of at least one six when 6 dice are rolled simultaneously.

(b) The appearance of at least two sixes when 12 dice are rolled simultaneously.

(c) The appearance of at least three sixes when 18 dice are rolled simultaneously.

4. Consider all the points (x, y) on the plane, where x, y are real numbers. The usual way of measuring the distance between two points (x_1, y_1) and (x_2, y_2) is given by the distance formula $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Let us consider different ways of measuring distance: For a real number $p \geq 1$, we define the p-distance between (x_1, y_1) and (x_2, y_2) by the formula

$$\left(|x_1 - x_2|^p + |y_1 - y_2|^p\right)^{1/p}.$$

(a) What is the shape of the set of all points (x, y) whose p-distance to $(0, 0)$ is less than or equal to 1? Consider the cases $p = 1$, $p = 3/2$, $p = 2$, $p = 3$ and $p = 10$.

(b) What happens as p tends to infinity? Can you find a formula for the ∞-distance between the points (x_1, y_1) and (x_2, y_2)?

5. What is the area enclosed by the paths of the two wheels when a bicycle of length a makes a 90 degree turn? (see the picture on the next page). Though we do not know a formula for the curves that form the paths, we can still compute this! Hint: Problem 3 in the previous set of problems will be helpful.

Bicycle turns from
one street to another