

Talent Search

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

https://www.ndsu.edu/math/ongoing_events/nd_talent_search/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by April 24, 2016, using the form on the website. Alternatively, solutions may be sent by email to maria.alfonseca@ndsu.edu, or by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the second set of problems:

1. The lateral surface area of a right circular cone is three times the surface area of its inscribed sphere. Find the angle α at the vertex of the cone (see figure).
2. An ant is walking along the edges of a cube. It starts out from vertex 1, and randomly takes one of the three edges that contain vertex 1. The probability of taking each edge is $1/3$. Each time it arrives to a new vertex, once again the ant continues its way along an edge containing that vertex, with probability $1/3$ (observe that there is a probability $1/3$ that the ant will go back along the last edge it came from). Assume that vertices 7 and 8 of the cube have been sprayed with insecticide (see figure).
 - (a) What is the probability that the ant will never reach one of those two vertices in her aleatory walk?
 - (b) What is the probability of its eventually reaching vertex 7?

- (c) How about of reaching vertex 8?
3. We have a piano with an unlimited number of keys. Considering only the white keys, we start by playing a C, then we play the next key (D), then we skip one and press F, then we skip two and play B, then we skip three and press F (which is on the next octave). We continue this process, always skipping one more key than in the previous step, until we have played 7000 keys.
- (a) How many times have we played a C?
- (b) Is there any note that we have never played?
4. In the island of knights and knaves, only these two kinds of people live. Knights always tell the truth, knaves always lie. You want to find the way to the city, and there are two possible roads in front of you, one due north and the other due east. You ask two locals which way leads to the city, and they reply the following: Person A: "If we are both knights, then the road due north leads to the city" Person B: "If only one of us is a knight, then the road due east leads to the city".
- (a) What kind of person is A?
- (b) Which road will take you to the city?
5. Prove that $4^n + 6n - 1$ is a multiple of 9 for every natural number n .

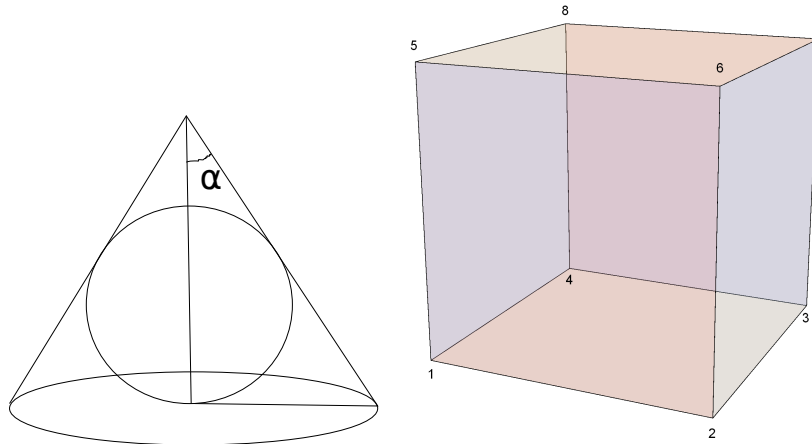


Figure 1: Figures for Problems 1 and 2