Talent Search

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

https://www.ndsu.edu/math/ongoing_events/nd_talent_search/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by February 11, 2017, using the form on the website. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the second set of problems:

1. Let n be an odd number. Let $a_1, a_2, \ldots, a_n$ be integers, and let $b_1, b_2, \ldots, b_n$ be the same numbers written in a different order. Show that
$$(a_1 + b_1)(a_2 + b_2) \cdots (a_n + b_n)$$
is an even number.

2. In the interior of a 60 degree angle we have a point M, such that the distances from M to each side of the angle are 2 cm and 11 cm, respectively. Find the distance from M to the vertex of the angle.

3. A snow avalanche traps a mole inside a snowball shaped like an ellipsoid with a volume of 500 cubic meters. The mole can dig through the snow advancing 1 meter per minute, but he has only strength and breath for 24 minutes. Can the mole reach the surface of the snowball and save its life?

4. Let O, A, B, C be four points, such that OA is perpendicular to OB, OB is perpendicular to OC, and OC is perpendicular to OA. We denote the length of OA by a, the length of OB by b, the length of OC by c. Show

   (a) All the angles of the triangle ABC are acute.

   (b) The line passing through O that is perpendicular to the plane containing ABC, intersects this plane at a point H which is the orthocenter of the triangle ABC.

   (c) What is the distance OH in terms of a, b, c?

5. An island is inhabited by two types of people: knights, who always tell the truth, and knaves, who always lie. In general, an inhabitant of the island does not know what kind each other is, only that he or she must belong to one of these two kinds. However, there are some individuals for which everybody knows they are knights (and they are called "established knights"), and there are also "established knaves", i.e. everybody knows they are knaves.

   There are several clubs in this island, and each person can belong to one or more of these clubs. Given a person X and a club C, either X affirms to belong to C, or affirms not to belong to C.

   The following rules are followed:

   - The group of all established knights is a club.
   - The group of all established knaves is a club.
   - Given any club C, the group of people who do not belong to C form a club, called $\overline{C}$.
   - Given any club C, there is at least one person X in the island who affirms to belong to C (however, this affirmation may be false if X is a knave).

   With this information, answer the following:

   (a) Prove that there is at least one non-established knight on the island.

   (b) Prove that there is at least one non-established knave on the island.

   (c) Does the group of all knaves form a club?

   (d) Does the group of all knights form a club?