Talent Search

The Department of Mathematics at NDSU is happy to announce the third round of the 2016-17 North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems and to reward these students for their efforts.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

https://www.ndsu.edu/math/ongoing_events/nd_talent_search/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by April 1, 2017, using the form on the website. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the third set of problems:

1. Starting from a square piece of paper, one can fold and cut to obtain rectangles of specific ratios, as in Figure 1. In each case, the rectangle we want is the one on the right side of the last step of the folding process.

   Show that the rectangle obtained in a.4 has the golden ratio $1 : (1+\sqrt{5})/2)$, the rectangle obtained in b.3 has ratio $1 : \sqrt{3}$, and the rectangle in c.3 has ratio $1 : \sqrt{2}$.

2. Every day at noon, a ship leaves from Le Havre in France towards New York, and at the same time a ship leaves New York bound to Le Havre. The crossing takes 7 days and 7 nights. If a ship leaves Le Havre today at noon, how many ships coming from New York will it meet?

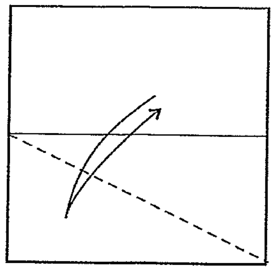
3. Let x, y be two real numbers such that $x+y=2$. Show that $x^2+y^2 \geq 2$ and $x^3+y^3 \geq 2$.

4. A **bounded closed interval** of the real line is a set $[a, b]$ consisting on all the real numbers x such that $a \leq x \leq b$, where a, b are some fixed real numbers.

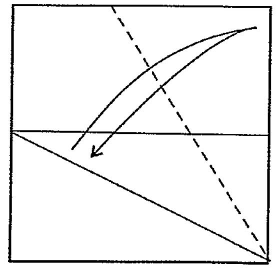
   Suppose that we are given n bounded and closed intervals on the real line, such that given any two of them, they have a point in common. Show that there is a point that belongs to all the given intervals.

5. We have a cylindrical mirror with center at the origin and radius 1. A point A on the x-axis is reflected into this mirror, and to our eyes, the reflection point Q appears to be "inside" the cylinder, also on the x-axis. See Figure 2. To find out this point Q, we consider a ray leaving from the point A and being reflected by the mirror at the point P. As in any mirror, the angle of incidence θ equals the angle of reflection, and if we prolong the reflected ray, it meets the x-axis on a point Q_φ. The reflection point Q of A can be approximated by Q_φ when the angle φ tends towards 0. Find the position of Q as a function of A, and also determine what happens to Q as A moves towards infinity along the x-axis (i.e., where we would see the reflection of a ray coming from very far, like from the Sun.)

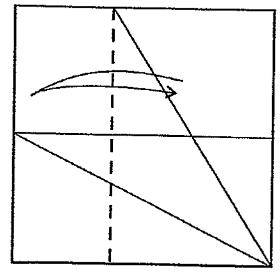
a.1



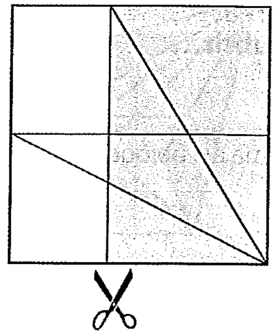
a.2



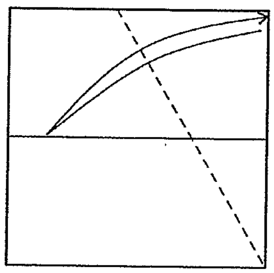
a.3



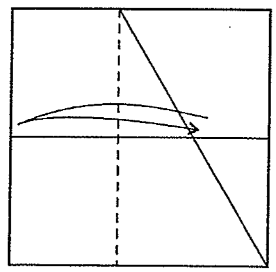
a.4



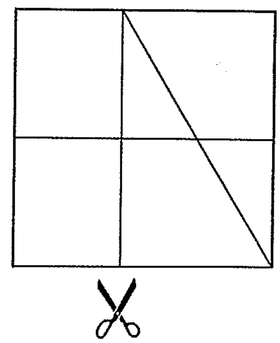
b.1



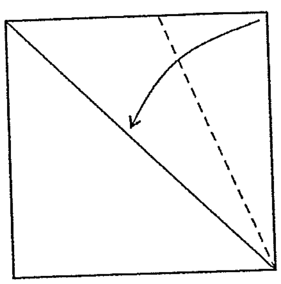
b.2



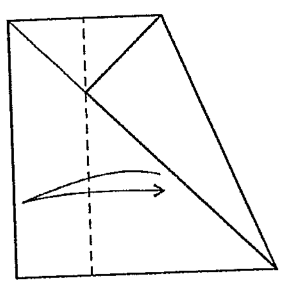
b.3



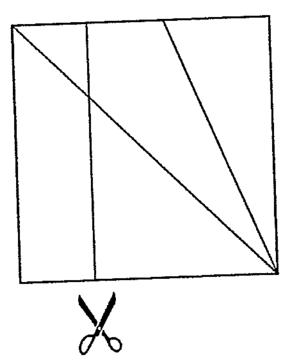
c.1



c.2



c.3



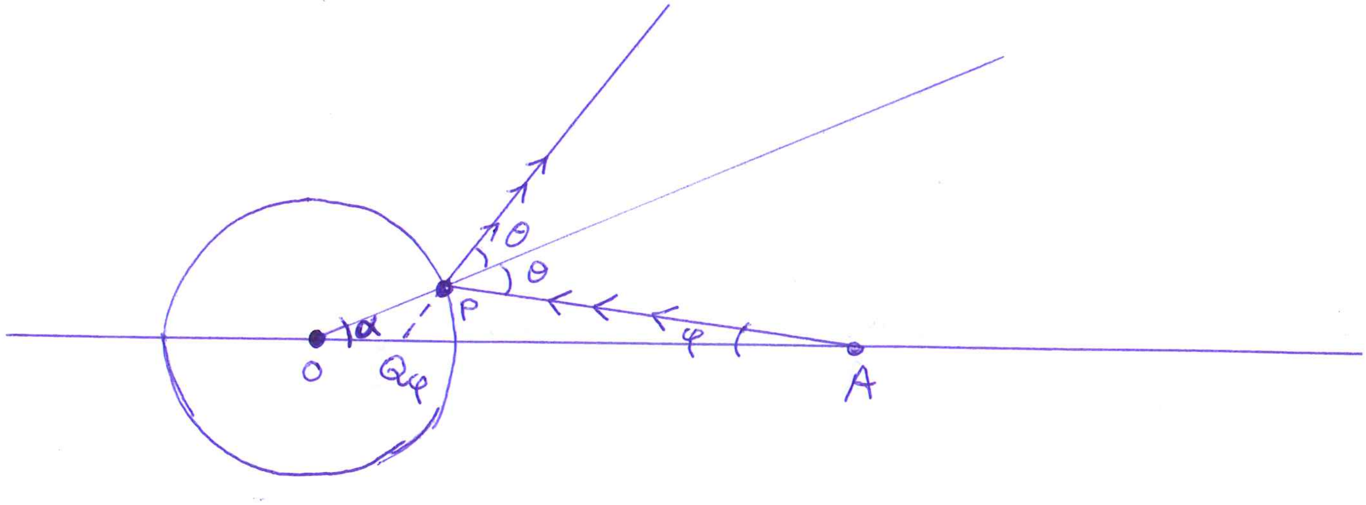


FIGURE 2