## Talent Search

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at
https://www.ndsu.edu/math/ongoing_events/nd_talent_search/
Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; finding a good solution to a problem is always an achievement. The problems do not require advanced mathematical knowledge - just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by March 31, 2018, using the form on the website. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.\# 2750
PO BOX 6050
Fargo, ND 58108-6050
Please do not forget to include your name, postal address, school, and e-mail address.

Here is the third (and last) set of problems:

1. $N$ balls of different colors are placed on a circle. At time 0 , all balls start moving along the circle at constant speed (same speed for all balls). Whenever two balls meet, they bounce (each ball reverses direction and continues moving along the circle at the same speed). Show that there is a time $T$ at which all the balls are back in their original positions.
2. A mouse is eating a cube of cheese that has been cut into 27 unit cubes. When the mouse finishes one unit cube, it moves on to a cube that shares a face with the cube it just finished eating. Is it possible for the mouse to eat everything except the central unit cube?
3. We have $N$ lines on a plane. No two lines are parallel, and no three lines pass through the same point. In how many regions do these lines divide the plane?
4. A metal bar is shaped like a triangular prism with base $A B C$ and volume $V$. Let us call $B B^{\prime}$ and $C C^{\prime}$ the two medians of the triangle passing through $B$ and $C$, and $G$ their point of intersection. We cut out the metal bar to obtain another triangular prism with base $B^{\prime} G C^{\prime}$. What is the volume of this new bar in terms of $V$ ?
5. A map of the southern hemisphere of the earth is constructed as follows: Place paper on a table, and a globe with the south pole touching the paper, and imagine that there is a light source at the center of the earth. Then meridians are projected into straight lines passing through the south pole (rays), and parallels are projected onto concentric circles on the paper. If the latitude of a parallel is $\ell$, and the radius of the globe is $R$, what is the radius of the projection of the parallel? Show also that this map does not preserve distances.
