The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search.The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

https://www.ndsu.edu/math/ongoing_events/nd_talent_search/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by January 31, 2022, using the form on the website. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the second set of problems:

1. Consider the set S of 3 digit numbers with no repetitions that can be made using only the digits $\{0, 1, 3, 4, 7, 8\}$. Having 0 as first digit is allowed. Some examples of numbers in the set S are 001, 013, 410, 784, etc.

   If we pick at random a number from S, what is the probability that it is a multiple of 3?

2. We consider two non-constant polynomials $P(x)$ and $Q(x)$, with the property that for every real number x,
$$P(Q(x)) = P(x)Q(x).$$
   What is the value of $Q(1)$?

3. Some sequences of numbers have a limit (approach a value), such as the sequence
$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$$
   where the numbers approach 0. Other sequences do not have a limit, like the sequence
$$0, 1, 2, 0, 1, 2, 0, 1, 2, 0, 1, 2, \ldots$$
   Let us call a general sequence $\{a_1, a_2, a_3, a_4, \ldots\}$. We define a new sequence in the following way:
$$s_1 = a_1, \; ; s_2 = \frac{a_1 + a_2}{2}, \; s_3 = \frac{a_1 + a_2 + a_3}{3} \ldots$$
   As you see, the term s_n of the new sequence is formed by averaging the first n terms of the original sequence. So, we call the sequence $\{s_n\}$ the "average sequence".

(a) If the original sequence is $0, 1, 2, 0, 1, 2, 0, 1, 2, \ldots$, what is its average sequence, and does it have a limit?

(b) Write another sequence that does not have a limit but whose average sequence does have a limit.

(c) Can you state a general rule as to what the limit of the average sequence is?

4. Consider a quadrilateral that does not have any pair of parallel sides. Divide each of the 4 sides into n equal segments (of course, the segments corresponding to different sides may have different lengths). Then connect the division points on opposite sides and color them black or white to create a checkerboard pattern. Show that if n is even, the sum of the areas of the black pieces is the same as the sum of the areas of the white pieces. (See left picture)

5. A double-W shape on the xy-plane is formed by the line segments from $\left(\frac{1}{2\sqrt{3}}, \frac{1}{2}\right)$ to $\left(\frac{1}{\sqrt{3}}, 0\right)$, and from $\left(\frac{1}{2\sqrt{3}}, \frac{1}{2}\right)$ to $\left(\frac{1}{\sqrt{3}}, 1\right)$, along with their reflection on the x-axis, and the reflection of the whole W on the y-axis.

   Assume that a light ray rises vertically from below, hits the W-shape and gets reflected following the Physics Law of Reflection (the angle of incidence equals the angle of reflection). Complete the trajectory of the light ray till it exists the double W shape. (See right picture)