

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

[https://www.ndsu.edu/math/ongoing\\_events/nd\\_talent\\_search/](https://www.ndsu.edu/math/ongoing_events/nd_talent_search/)

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

**Please upload and submit your solutions by April 15, 2022, using the form on the website.** Alternatively, solutions may be sent by regular mail to:

Talent Search  
c/o Maria Alfonseca  
Mathematics NDSU Dept.# 2750  
PO BOX 6050  
Fargo, ND 58108-6050

**Please do not forget to include your name, postal address, school, and e-mail address.**

Here is the third set of problems:

1. A candy bowl has the following candies in it: 2 cherry, 3 apple, 4 grape, and 5 lemon. You choose 4 candies from the bowl. How many different candy combinations are there?
2. The first 60 positive integers are partitioned in 30 pairs, so that the difference between the two numbers in the same pair is either 1 or 10. Given that (10,11) and (20,30) are two of the pairs, which number is paired with 41? Explain your answer.
3. There is an island where each inhabitant is either a knight or a knave. Knights always say true statements, and Knaves always say false statements. In addition, each person is either sane or crazy. Sane people believe that true statements are true, and false statements are false; while crazy people believe that true statements are false, and false statements are true. Hence, we have four types of people in this island:
  - Sane knights, that utter only true statements.
  - Crazy knights, that utter only false statements, because they think that they are true.
  - Sane knaves, that utter only false statements.
  - Crazy knaves, that utter only true statements, because they think that they are false.

It makes a great difference for a local to say the statement “Fact  $X$  is true” versus the statement “I believe that fact  $X$  is true”. Can you explain what the difference is?

4. A **domino tiling** of an  $m \times n$  chessboard is a covering of a grid of squares by dominoes such that each domino covers exactly two squares of the chessboard and no dominoes are overlapping.
- (a) There are 2 distinct domino tilings of a  $2 \times 2$  chessboard (both dominoes horizontal or both vertical). How many distinct domino tilings are there of a  $2 \times 3$  chessboard? A  $2 \times 4$  chessboard? A  $2 \times 5$  chessboard? How would you calculate the number of tilings of a  $2 \times n$  chessboard?
  - (b) Is it possible to create a domino tiling of an  $8 \times 8$  chessboard in which two squares at opposite corners have been removed? Explain your answer.
5. Consider an infinite sequence of non-negative real numbers,  $(x_1, x_2, x_3, x_4, \dots)$ .
- (a) Assume that the sum of the sequence is finite,  $\sum_{i=1}^{\infty} x_i < \infty$ . Does it follow that  $\sum_{i=1}^{\infty} x_i^2 < \infty$ ? Show that it is true or provide a counterexample.
  - (b) How about the converse statement? If we know that  $\sum_{i=1}^{\infty} x_i^2 < \infty$ , does it follow that  $\sum_{i=1}^{\infty} x_i < \infty$ ? Show that it is true or provide a counterexample.