Problems for Preliminary Exam
Applied Mathematics, ODE
August 2023

- **Answer any 6 problems. If you attempt all the problems, clearly indicate which 6 you want to be graded. Otherwise the first 6 will be graded.**

1. Solve the equation

$$t(e^y - y') = 2.$$

2. Find a periodic solution of equation

$$y' = 2y\cos^2 t - \sin t.$$

3. Show that only one solution of equation

$$ty' - (2t^2 + 1)y = t^2$$

tends to a finite limit as $t \to \infty$. Find this limit.

4. Find all real numbers a, b such that all solutions of equation

$$y'' + ay' + by = 0$$

are bounded on $(-\infty, \infty)$.

5. Find the Green's function for the problem

$$y'' - y = f(t),$$

$y(\cdot)$ is bounded as $t \to \pm\infty$.

6. Does there exist a solution of the initial value problem

$$y' = f(y), \qquad f(y) = \begin{cases} 1, & \text{if } y < 0, \\ -1, & \text{if } y \geq 0, \end{cases} \qquad y(0) = 0?$$

7. Does the equation

$$\ddot{x} - 2\dot{x} = 8\sin^2 t$$

have a periodic solution?