• Answer any 6 problems. If you attempt all the problems, clearly indicate which 6 you want to be graded. Otherwise the first 6 will be graded.

1. Find a solution of equation
   \[ \ddot{x} + x = x^2 \]
   that is decreasing and tending to one as \( t \to \infty \).

2. Solve the equation
   \[ (t^2 + y^2 + t)dt + ydy = 0. \]

3. For which integer \( b \) and \( c \) the equation
   \[ y''' + b^2 y' = \sin t + c \sin^2 t \]
   has a periodic solution?

4. Consider the equation
   \[ ty' + ay = f(t), \]
   where \( a \) is a positive constant, \( f(t) \to b \) as \( t \to 0 \). Prove that there exists a unique solution \( y \) that is bounded as \( t \to 0 \). Find the limit of this solution at zero.

5. For which \( n \) there exist continuous functions \( p_0, \ldots, p_{n-1} \) such that equation
   \[ y^{(n)} + p_{n-1}(t)y^{(n-1)} + \ldots + p_0(t)y = 0 \]
   has a solution \( y(t) = t^3 \).

6. Is it true that zero solution of system
   \[ x' = x - y, \quad y' = 5x - 5y \]
   is asymptotically stable?

7. Solve the initial value problem
   \[ (t + 2y)y' = 1, \quad y(0) = -1. \]