Problems for Preliminary Exam
Applied Mathematics, ODE
January 2024

- **Answer any 6 problems. If you attempt all the problems, clearly indicate which 6 you want to be graded. Otherwise the first 6 will be graded.**

1. Find a solution of equation

$$\ddot{x} + x = x^2$$

that is decreasing and tending to one as $t \to \infty$.

2. Solve the equation

$$(t^2 + y^2 + t)dt + ydy = 0.$$

3. For which integer b and c the equation

$$y''' + b^2 y' = \sin t + c \sin^2 t$$

has a periodic solution?

4. Consider the equation

$$ty' + ay = f(t),$$

where a is a positive constant, $f(t) \to b$ as $t \to 0$. Prove that there exists a unique solution y that is bounded as $t \to 0$. Find the limit of this solution at zero.

5. For which n there exist continuous functions $p_0, \ldots, p_{n-1}$ such that equation

$$y^{(n)} + p_{n-1}(t)y^{(n-1)} + \ldots + p_0(t)y = 0$$

has a solution $y(t) = t^3$.

6. Is it true that zero solution of system

$$x' = x - y, \qquad y' = 5x - 5y$$

is asymptotically stable?

7. Solve the initial value problem

$$(t + 2y)y' = 1, \qquad y(0) = -1.$$