Instructions. Answer **any 4 short questions, and any 4 long questions**. Clearly mark which questions you wish to be graded on this sheet, or else 1-4 and 6-9 will be graded. Show all work, and explain your answers clearly. Answers will be graded on correctness and clarity. **All solutions must include explanation.**

Shorter questions: (5 points each)

1. Let $\mathbf{n}$ denote an antichain poset with n elements. Find a formula for the number of order ideals of the ordinal sum poset $\mathbf{n} \oplus \mathbf{m}$.
2. Compute the Möbius function of the poset $a \lessdot b \lessdot e, a \lessdot c \lessdot e$, $a \lessdot d \lessdot e$.
3. Suppose P and Q are semistandard Young tableaux. What additional properties would P and Q need to satisfy so that applying the inverse Robinson-Schensted-Knuth algorithm to (P, Q) results in an *involution* $\pi \in S_n$?
4. Find the cycle index polynomial $Z(G)$ for the cyclic group $G = \langle(1, 2, 3, 4)\rangle$ acting on the set $X = [4]$ (i.e. the group of rotations of a square).
5. Pick your favorite basis of Sym_3, the ring of homogeneous symmetric functions of degree 3 over $\mathbb{Q}$. Write all these basis polynomials in the variables x_1, x_2, x_3.

Longer questions: (10 points each)

6. Let Z_n denote the "zig-zag" poset with elements $x_1, x_2, \ldots, x_n$ and cover relations $x_{2i-1} \lessdot x_{2i}$ and $x_{2i} \gtrdot x_{2i+1}$. Let $A(n)$ denote the number of antichains of Z_n. Find $A(n)$ for $n = 1, 2, 3, 4$. Then find and prove a recursive formula for $A(n)$.
7. State the fundamental theorem of finite distributive lattices. Give an example of the correspondence in this theorem and outline the key parts of the proof.
8. Verify that $(X, \langle g\rangle, f(q))$ exhibits the cyclic sieving phenomenon for X the set of 4-digit binary words with two zeros and two ones, g acting on X by a cyclic shift, and $f(q) = 1 + q + 2q^2 + q^3 + q^4$.
9. Find at least three columns of the table of characters for the irreducible representations of S_4. Make sure to appropriately label the rows with partitions and columns with conjugacy classes. Give reasons for each character value; writing the table from memory without justifying your computations will receive no credit.
10. Write the basis element of the Specht module corresponding to the standard Young tableau $T = \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & 5 & 6 \\ \hline\end{array}$. Then outline the proof of ONE of the following theorems:
    - The Specht modules indexed by partitions of n are the irreducible representations of the symmetric group S_n, OR
    - The set of polytabloids indexed by standard Young tableaux is a basis for the Specht module S^λ.