Please write all answers using a blue pen. Only write on one side of the paper. Clearly cross-out any work which you do NOT want us to grade.

(1) Let F be a compact, connected surface (orientable or non-orientable), with a single boundary component C. Show that there is no retraction from F onto C.

(2) Let $X = (S^1 \times S^1)/\{p,q\}$ be a torus with 2 points removed and $Y = \mathbb{R}P^2/\{a,b,c\}$ the projective plane with 3 points removed. Prove or disprove: X and Y are homotopy equivalent.

(3) Let T be a torus and C a homotopically non-trivial, simple closed curve in T. Let X be the 2-complex obtained by attaching a 1-punctured torus , S, to T along C. (that is by identifying ∂S with C).
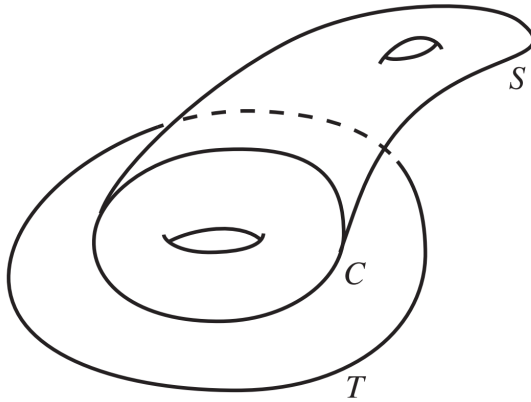
$X = T \cup_C S$

(a) Compute $\pi_1(X)$.
(b) Show that C must lift to a closed loop (as opposed to an open path) on any 2-fold cover of X.
(c) Give three non-homeomorphic, connected, covering spaces of X and exhibit (explain or draw) a covering map for each. You needn't show that the spaces are not homeomorphic. Are there any others?