

# Math 747, Differential Topology Preliminary Exam

January 2024

**Problem 1.** a) State Stokes Theorem.

b) Let  $M$  be a smooth  $n$ -manifold with boundary  $N$  and  $\xi$  be a closed differential  $(n - 1)$ -form on  $M$ . Show that  $\int_N \xi = 0$ .

**Problem 2.** Let  $M$  be a smooth manifold. Show that the tangent bundle  $TM$  is an orientable manifold.

**Problem 3.** a) State Whitney Embedding Theorem.

b) State Massey Immersion Theorem.

c) Is it true that all smooth compact connected 2-manifolds immerse in  $\mathbb{R}^3$ ?

**Problem 4.** Let  $c$  be a real number and  $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a map given as  $F(x, y) = (x^3 + 2024y, cy^3 + x)$ . For what values of  $c$  is  $F$

a) an immersion?

b) an embedding?