Thermal Transients in Closed, Unsaturated Soil Systems

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ABSTRACT

Soil heat and unsaturated water transport are coupled processes. Philip and de Vries (PdV) theory has been widely applied but some aspects of it are being questioned. This study uses SPLaSHWaTr 2.4 to simulate transients in closed, 5-cm long sand and silt loam columns. Three temperature boundary condition types are considered: rise and plateau (r), periodic (p) cycling, and transition (t) between steady states. Results of r for loam appear similar to experimental data cited by PdV. The p condition results in a persistent drier region near the cycled boundary. The simulations provide insight into the design of new experiments. Steady state was attained in one and six days for sand and loam, respectively, with t, while with r attainment took up to twice as long.

1. Introduction

In moist soil, temperature changes induce water movement and water content influences heat transport. A detailed, mechanistic model (Philip and de Vries, 1957; de Vries, 1958), henceforth PdV, formulates transport coefficients and their dependence on soil properties.

Simulators of heat dissipation from underground nuclear facilities and electrical cables have invoked PdV (Anders and Radhakrishna, 1988; Hartley, 1977; Radhakrishna et al., 1984). Necessary inputs for global atmospheric models include land surface heat and water vapor fluxes, calculable by PdV (Milly and Eagleson, 1982). Desert edaphic studies (Scanlon and Milly, 1994) and agricultural soil management (Benjamin et al., 1990) have benefited from PdV models.

Recent literature, however, finds a need for closer inspection. Schelde et al. (1998) say the vapor flow enhancement of PdV is difficult to estimate. Salzmann et al. (2000) limited their consideration to simulations because there have been too few column experiments for sound model testing. Bach (1992) found that a commonly used temperature coefficient of matric potential was too large by a factor of three for very wet soil. Webb (1999), contrary to PdV, contends the vapor diffusion enhancement factor applies to concentration and temperature gradients. Parlange et al. (1998) proposed that diurnal expansion of soil air accounts for field experiment vapor flux higher than PdV predictions.

These suggested deviations from PdV theory call for new work. The literature on PdV is extensive, but does not graphically present full transient simulation of prototypical experiments. Simulation at the laboratory scale would be desirable for planning new experiments. Typical experiments have been conducted on sealed soil columns initially at uniform temperature and moisture content. At time \( t = 0 \) new end temperatures are imposed and maintained indefinitely. Water then redistributes and nonequilibrium temperatures develop. Data from such experiments (Taylor and Cavazza, 1954) were cited by PdV in the original work. Others, including Cassel et al. (1969), later did similar experiments.

Our objective is to present transient PdV water content and temperature simulations for experiments as described above, and also for experiments using cycling and transition boundary temperatures. This manner of presenting results is analogous to transient moisture content profiles commonly used to portray infiltration. We haven’t found similar presentation of corresponding heat-moisture problems. Our simulated PdV results can provide benchmarks for designing new experiments that may lead to a refined theory.

2. Materials and Methods

We used the Simulation Program for Land-Surface Heat and Water Transport (SPLaSHWaTr) version 2.4, henceforth called SPL. It uses the PdV approach but with matric head as an independent variable, rather than moisture content. Several publications (Milly and Eagleson, 1980, 1982; Milly, 1982; Milly, 1984; Milly, 1985) document SPL. Additional work has applied SPL to simulation of laboratory experiments (Bach, 1992) and long-term moisture movement in a desert environment (Scanlon, 1990; Scanlon and Milly, 1994).

The SPL model considers media that is heterogeneous, non-deforming, isotropic, and hysteretic (Milly and Eagleson, 1980). Within this context, SPL numerically solves the continuity equations

\[
\frac{\partial Q_i}{\partial t} = \frac{\partial J_i}{\partial z} + S_i \quad i = 1, 2 \quad [1]
\]

where \( Q \) is quantity per unit volume, \( J \) is flux (quantity/area/time), \( S \) is source strength, \( t \) is time, \( z \) is vertical distance, and \( i \) represents water or energy. Coupling results because the \( Q_s \), \( J_s \), and \( S_s \) are functions of the same state variables, \( \psi \) and \( T \), where \( \psi \) is matric head and \( T \) is temperature. Incorporation into (1) of specific transport and capacity relations for liquid water, water vapor, and heat results (Milly and Eagleson, 1980, p. 99, 100) in

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\[ c_1 \frac{\partial \psi}{\partial t} + c_2 \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( c_3 \frac{\partial \psi}{\partial x} + c_4 \frac{\partial T}{\partial x} + c_5 \right) + c_6 \]  

[2]

\[ d_1 \frac{\partial \psi}{\partial t} + d_2 \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( d_3 \frac{\partial \psi}{\partial x} + d_4 \frac{\partial T}{\partial x} + d_5 \right) \]  

[3]

where (2) is the mass conservation equation and (3) is the energy conservation equation. The \( c_i \)s and \( d_i \)s are functions of \( \psi \) and \( T \). For detailed descriptions of them see Milly (1984).

Model SPL was written to represent a soil column from the surface to beyond rooting depth. Dynamic surface boundary conditions are allowed but bottom end conditions may only be static. To simulate horizontal columns we removed gravity from SPL and replaced \( z \) of (1), (2), and (3) with \( x \).

Moisture initial and boundary conditions were

\[ \psi = \psi_0 \quad t < 0, \quad 0 \leq x \leq L \]  

[4a]

\[ c_3 \frac{\partial \psi}{\partial x} + c_4 \frac{\partial T}{\partial x} = 0 \quad t \geq 0, \quad x = 0, L \]  

[4b]

with (4b) specifying a closed system. Temperature initial and boundary conditions (Fig. 1) were \( r \) (rise to plateau), \( p \) (periodic triangular), or \( t \) (transition to reversed steady state). All runs modeled a region of \( L = 5 \) cm with 100 equal-length elements. Soil data for sand and silt loam soil were from Milly and Eagleson (1982), Table 2.2.

Our procedure to verify satisfactory convergence followed the general scheme of Milly and Eagleson (1982). That is, we compared results using our standard number of nodes (101) and maximum allowed changes per time step for temperature (0.1 °C) and volumetric moisture (0.001) to results using half the nodes (51) and larger time steps defined by allowed changes five times greater per step.

### 3. Results

#### 3.1 Convergence

Convergence was satisfactory. Graphs of temperature and moisture content were almost identical between runs with standard nodes and time steps versus runs with the larger nodes and time steps. In addition, all the convergence test runs used 1.0 °C/min as the boundary temperature change rate versus 0.25 °C/min maximum for all simulations below.

#### 3.2 Transient Temperature and Moisture

Results at sequential times are summarized graphically (Fig. 2), indicating times, initial conditions, and temperature boundary conditions (bc).

Boundary condition \( r \) (Fig. 1) caused drying which progressed from the warm end toward the center while wetting advanced from the cool end (Fig. 2(a, b, c, g, h, and i)). After about one day for sand and six days for loam, a stable (steady-state) moisture profile of sigmoidal shape developed. It appears time to steady state is determined largely by the total water displacement.

Cyclical boundary conditions, \( p \) of Fig. 1, produce interesting results (Fig 2 (j, k, m, and n)). The first and last lines in the sequence, one full cycle apart, overlap almost exactly in every case. This indicates that after 9 cycles all noncyclical transient behavior has decayed away. In all instances the region near the end subject to the temperature cycling develops a lower than average moisture content. This drier region bottoms out (k and m) or slopes (j and n). Simulations as above except with a no heat flow condition at \( x = 0 \) produced moisture profiles (no figure) which were nearly constant except very near \( x = 0.05 \).

Boundary condition \( t \) (Fig. 1) produces a steady-state to opposite steady-state transition. The initial steady state condition is the result of applying nonequilibrium temperatures (20 °C left, 35°C right) for several days (see Fig. 2) before \( t = 0 \). The right-end temperature starts transition from 35 °C to 5 °C at \( t = 0 \), completes at \( t_t \), and remains at 5 °C thereafter. Time to arrive at opposite steady state is slightly greater than to reach steady state using \( r \).

### 4. Discussion and Conclusions

Boundary condition \( r \) is the prototype of numerous experimental studies of coupled heat and moisture flow in unsaturated soil. However, transient simulation model results similar to Fig. 2 panels with \( bc = r \) have not previously been reported. Experimental results for a loam soil with \( r \) type boundary conditions were reported by Taylor and Cavazza (1954) and were referenced by Philip and deVries (1957) in developing their theory. Taylor and Cavassa (1954, Fig. 1) experimental results at
steady state were like Fig. 2(g) for silt loam soil at similar moisture. However, steady state was reached in the 10-cm long experimental columns in 4 d compared to 24 d expected for a 10-cm long column corresponding to Fig. 2(g).

Also, temperature profiles (Fig. 2(d, e, and f)) agree with Prunty and Horton (1994) in that they are concave upward. Earlier widely cited experimental observations of concave down temperature profiles were in error because of ambient temperature interference (Prunty and Horton, 1994).

With cyclcical temperatures (bc = p) persistent moisture profiles (Fig. 2(j, k, m, and n)) develop as noncyclic transients decay. The general effect is consistent with reduced transport capacity in drier material, but needs detailed experimental verification. Such experiments may provide an important and possibly more sensitive test of the theory.

Simulations of bc = t (Fig. 2(l and o)) also suggest potential future experiments. An advantage of the p and t bc is that they do not depend on the experimentally problematic uniform initial water content condition.
However, they would be more complex to implement experimentally. Improvements in experimental techniques are needed for better resolution, reproducibility, and model testing. At roughly 50 years after the experiments on which PdV theory was based there is not yet a data base of column experiment results similar to the simulations results presented here. Simulations can serve, however, as useful guides to the design of new, more productive column experiments. Further, experiments in short columns are important because of time. An experiment completed in time $t$ in a laboratory column 0.05 m long requires $(2.5/0.05)^2 \tau = 2500 \tau$ to complete in a column 2.5 m long.

References


