Lesson Title:
Probability and Odds

Topic:
A presentation and hands on activity for students which covers theoretical probability, experimental probability, and odds.

Content Level:
Mathematics

Time Requirement:
Two class periods, plus homework and posttest.

Materials:

- *PowerPoint (or Keynote)*
- *Overhead Projector*
- *A penny for each student to flip*
- *A die for each student to roll*

Background:
From knowing the chance it will snow on a given day to knowing the likelihood of winning millions playing the lotto to knowing the chance one will get a certain disease or cancer, we encounter probability and odds everyday. It is important that we understand the different types of probability and odds as well as their relationship to each other. Knowing how to use these mathematical tools and being able to recognize when these tools have been used incorrectly is vital.

The theoretical probability of an event E is the ratio of ways the event can occur to the total number of possibilities in the sample space. Recall that the sample space is the set of all possible outcomes for an event. To compute the theoretical probability of an event E, we use the following formula

$$P_t(E) = \frac{\text{number of ways event can occur}}{\text{total number of possibilities}}$$

The experimental probability of an event E is the ratio of the number of times the event occurs to the total number of trials. To compute the experimental probability of an event E, we use the following formula

$$P_e(E) = \frac{\text{number of times event occurs}}{\text{total number trials}}$$

It is important to note that as the number of trials increases, the experimental probability will approach the theoretical probability.

The odds of an event E is the ratio of the number of ways the event can happen (number of successes) to the number of ways the event cannot happen (number of failures). To compute the odds of an event E, we use the following formula

$$\text{odds}(E) = \frac{\text{number of successes}}{\text{number of failures}}$$

Skills:

- *Collecting data*
- *Applying equations*
- *Simplifying fractions*

Objectives:

- *Students will learn about sample space.*
- *Students will learn how to compute the theoretical probability of a given event.*
- *Students will learn how to compute the experimental probability of a given event.*
- *Students will lean how to compute the odds of a given event.*
- *Students will discover the difference between theoretical and experimental probability.*
- *Students will be able to distinguish between probability and odds.*
- *Students will discover that when computing experimental probability, as the number of trials increases, the experimental probability will approach the theoretical probability.*

Standards Addressed:

- *Minnesota Comprehensive Assessments Series II, 2005*
  - *II.A.2*
  - *IV.B.1*
  - *IV.B.2*
  - *IV.B.3*

Procedure:

1. *Give the pretest.*
2. *Give a lecture/presentation on the concepts of sample space, theoretical probability, experimental probability, and odds. (see attached PowerPoint presentation)*
3. *As homework, give students a worksheet. (see attached worksheets; students had a lot of difficulty with the first worksheet so we threw that out and gave them the second worksheet).*
4. *Give the posttest shortly after the completion of the lecture/presentation and homework.*

Pre & Post Test Solutions:

Pretest:	*Posttest:*
1. *T*	1. *F*
2. *F*	2. *T*
3. *F*	3. *F*
4. *C*	4. *D*
5. *A*	5. *A*
6. *D*	6. *C*
7. *B*	7. *A*
8. *A*	8. *B*

Acknowledgements:

This activity was developed by Diana Beck, GraSUS - II Fellow. Support was provided by the Center for Science and Math Education and the GraSUS project: The Graduate Student-University-School (GraSUS) Collaborative for Science, Engineering, and Technology is funded by National Science Foundation GK-12 Program Grant DGE-0338128 to North Dakota State University. Special thanks are given to Lisa Erpelding for her support, encouragement, and time.

Name _____

Experimental Probability, Theoretical Probability, and Odds Pre-test

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True / False

_____ 1. Probability is a way to describe the likelihood of an event occurring.

_____ 2. Experimental probability is always more accurate than theoretical probability.

_____ 3. The odds of an event occurring is exactly the same as the probability that the event will occur.

Multiple Choice

_____ 4. If I were to roll a die exactly one time, the theoretical probability of rolling a 6 is (a) 1/2, (b) 1, (c) 1/6, or (d) 2/3.

_____ 5. If I were to randomly draw a card out of a regular deck of cards, what is the theoretical probability that the card I drew belonged to a red suit? (a) 1/2, (b) 3/26, (c) 1/ 52, or (d) 2/52

_____ 6. Suppose that I flipped a penny 5 times and that my results of each flip were as follows: flip 1 – H, flip 2 – H, flip 3 – T, flip 4 – H, flip 5 – T. Then the experimental probability of getting a H is (a) 1, (b) 1/2, (c) 2/5, or (d) 3/5.

_____ 7. Suppose that I roll a die exactly one time. The odds of rolling a 2 is (a) 1/6, (b) 1/5, (c) 1, or (d) 1/3.

_____ 8. Suppose that the numbers 1 through 45 are painted on purple ping-pong balls; and suppose that I place these balls in a bag. If you draw one ball at random from the bag, what are the odds that the number on your ball is odd? (a) 23/22, (b) 1/2, (c) 1/45, or (d) 1

Name _____

Experimental Probability, Theoretical Probability, and Odds Post-test

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True / False

_____ 1. The odds of an event occurring is exactly the same as the probability that the event will occur.

_____ 2. Probability is a way to describe the likelihood of an event occurring.

_____ 3. Experimental probability is always more accurate than theoretical probability.

Multiple Choice

_____ 4. Suppose that I flipped a penny 5 times and that my results of each flip were as follows: flip 1 – H, flip 2 – H, flip 3 – T, flip 4 – H, flip 5 – T. Then the experimental probability of getting a H is (a) 1, (b) 1/2, (c) 2/5, or (d) 3/5.

_____ 5. If I were to randomly draw a card out of a regular deck of cards, what is the theoretical probability that the card I drew belonged to a red suit? (a) 1/2, (b) 3/26, (c) 1/ 52, or (d) 2/52

_____ 6. If I were to roll a die exactly one time, the theoretical probability of rolling a 6 is (a) 1/2, (b) 1, (c) 1/6, or (d) 2/3.

_____ 7. Suppose that the numbers 1 through 45 are painted on purple ping-pong balls; and suppose that I place these balls in a bag. If you draw one ball at random from the bag, what are the odds that the number on your ball is odd? (a) 23/22, (b) 1/2, (c) 1/45, or (d) 1

_____ 8. Suppose that I roll a die exactly one time. The odds of rolling a 2 is (a) 1/6, (b) 1/5, (c) 1, or (d) 1/3.

Name _____

Experimental Probability, Theoretical Probability, and Odds

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1. In class, we flipped a coin 5 times and computed the experimental probability of getting tails. Then we flipped a coin 25 times and computed the experimental probability of getting tails. Now, flip a coin 50 times, recording whether you got heads or tails at each flip in the table below, and then compute the experimental probability of getting tails when you flip a coin.

Flip 1 -	Flip 11 -	Flip 21 -	Flip 31 -	Flip 41 -
Flip 2 -	Flip 12 –	Flip 22 -	Flip 32 -	Flip 42 -
Flip 3 -	Flip 13 -	Flip 23 -	Flip 33 -	Flip 43 -
Flip 4 -	Flip 14 -	Flip 24 -	Flip 34 -	Flip 44 -
Flip 5 -	Flip 15 -	Flip 25 -	Flip 35 -	Flip 45 -
Flip 6 -	Flip 16 -	Flip 26 -	Flip 36 -	Flip 46 -
Flip 7 -	Flip 17 -	Flip 27 -	Flip 37 -	Flip 47 -
Flip 8 -	Flip 18 -	Flip 28 -	Flip 38 -	Flip 48 -
Flip 9 –	Flip 19 -	Flip 29 -	Flip 39 -	Flip 49 -
Flip 10 -	Flip 20 -	Flip 30 -	Flip 40 -	Flip 50 -

P_e(Tails) =

Suppose we flipped a coin 100 times, 1000 times, 10,000 times, 100,000 times, and then 1,000,000 times. Also assume that after each round of flipping we calculated the experimental probability of getting tails. As the number of flips in each round increases, what do you think happens to the experimental probability of getting tails?

_____

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_____

2. Take a regular deck of cards and shuffle well. Draw one card from anywhere in the deck. Note whether this card is a black suit or a red suit in the table below. Put your card back in the deck and shuffle well. Draw another card from anywhere in the deck. Again note whether this card is a black suit or a red suit in the table below. If you repeat this process a total of 10 times, find the experimental probability that a card drawn from a regular deck of cards is a black suit.

1st draw -	3rd draw -	5th draw -	7th draw -	9th draw -
2nd draw -	4th draw -	6th draw -	8th draw -	10th draw -

P_e(black suit) =

3. In class we computed the sample space when we flipped a coin 3 times and then we computed the theoretical probability of having exactly one tail in those 3 flips. Now, write down the sample space when we flip a coin 4 times. Then compute the theoretical probability of having exactly 2 heads appear in those 4 flips.

   Sample Space:

$$P_t(2 \text{ heads}) =$$

4. In class we computed the sample space when we roll a die two times and then we computed the theoretical probability of having rolled two 1's. Now, write down the sample space when we roll a die 3 times. Then compute the theoretical probability of rolling a 2, 3, and 5 (not necessarily in this order).

   Sample Space:

$$P_t(\text{rolling a 2, 3, and 5}) =$$

5. Suppose that your roll 2 dice. What are the odds that you rolled doubles? Be sure to justify your answer.

6. If you shuffle a regular deck of cards and draw a card at random from the deck, what are the odds that the card you drew is a black queen? Be sure to justify your answer.

7. What is the difference between experimental probability and theoretical probability?

_____
_____
_____
_____
_____
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_____

8. Where might experimental probability, theoretical probability, and odds be used in the real world today?

_____
_____
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_____
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_____

9. How does the probability of an event occurring and the odds of an event occurring differ?

_____
_____
_____
_____
_____

Name _____

Probability and Odds

You must show all of your work! Solve the problems below and fill in missing letters correctly to answer the following question:

What did one math book say to the other math book?

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1. Suppose that Diana flips a coin 3 times.
   (a) Write down the sample space of flipping a coin 3 times:

   (b) What is the theoretical probability that Diana had exactly one head appearing in those 3 flips?

   P_t(one head)=_____

2. Suppose that you roll 2 dice. Calculate the odds that you rolled doubles.

   odds(doubles)=_____

3. Mary loves to play cards. After shuffling, she draws one at random. What is the theoretical probability that the card Mary drew was a seven?

   P_t(seven)=_____

4. Suppose that you shuffle a regular deck of cards and draw a card at random. What are the odds that the card you drew is a black queen?

   odds(black queen)=_____

5. Lisa has a spinner which has the numbers 1 through 7 equally spaced apart. What are the odds that she will get a prime number when spinning?

   odds(prime)=_____

6. Michael has a bag filled with ping-pong balls which have the numbers 2 through 21 painted on them. If he draws one ball at random from the bag, what is the theoretical probability that the number on the ball is even?

P_t(even)=_____

7. Danny has a bag filled with ping-pong balls which have the numbers 2 through 21 painted on them. If he draws one ball at random from the bag, what are the odds that the number on the ball is even?

odds(even)=_____

8. Jessica spins a spinner which has the numbers 1 through 5 equally spaced apart. On her first spin, Jessica gets a 2; on her second spin, Jessica gets a 5; on her third spin Jessica gets a 3; and on the fourth spin, Jessica gets a 5. What is the experimental probability of spinning a 2?

P_e(2)=_____

9. Anthony and his friend Jake play a guessing game. Anthony thinks of a number between 7 and 27 and then Jake tries to guess the number that Anthony is thinking of. What is the theoretical probability that the number Anthony's thinking of is 23?

P_t(23)=_____

10. Kevin has a container which has tennis balls that have the numbers 1 through 33 painted on them. If he draws a ball at random, what are the odds that the number on the ball is a multiple of 5?

odds(multiple of 5)=_____

11. Suppose you find a bowl filled with change. You determine that there are 73 pennies, 4 nickels, 19 dimes, and 4 quarters in the bowl. What is the theoretical probability of randomly selecting a penny?

P_t(penny)=_____

12. Suppose you find a bowl filled with change. You determine that there are 73 pennies, 4 nickels, 19 dimes, and 4 quarters in the bowl. What are the odds of randomly selecting a penny?

odds(penny)=_____

13. At a birthday party you are blindfolded. You are then told to randomly draw a treat from a bag which has 21 pencils, 13 erasers, and 11 packs of gum. What is the theoretical probability of randomly drawing an eraser?

P_t(eraser)=_____

14. Alex rolls a 7 sided die. What are the odds that he rolled a 2?

odds(2)=_____

15. You decide to go fishing one warm summer day. Suppose there are 10 walleye, 15 northern pike, 44 sunfish, and 11 bass in the lake. When you hook into a fish, what is the theoretical probability that you will catch a walleye?

P_t(walleye)=_____

16. You decide to go fishing one warm summer day. Suppose there are 10 walleye, 15 northern pike, 44 sunfish, and 11 bass in the lake. When you hook into a fish, what are the odds that you will catch a bass?

odds(bass)=_____

17. You decide to go fishing one warm summer day. Suppose there are 10 walleye, 15 northern pike, 44 sunfish, and 11 bass in the lake. If you have kept 2 sunfish, 1 bass, and 1 walleye, what is the theoretical probability that you will catch a northern pike?

P_t(northern pike)=_____

To fill in the missing letters, here's what you need to do. Suppose that the answer to question 1 was 4/3. Since Y=4/3 below, everywhere you see a 1 in a box below, you should put a Y in the same box.

B=3/8	D=73/27	E=1/25	G=11/69	H=1/6	I=1/20
L=1/2	M=1/8	N=1/5	O=2/9	P=73/100	R=1/4
S=1/13	T=1	V=15/76	W=13/45	Y=4/3	

12	10	2	'	7		1	10	6	14	4	8
	15	4		9	'	17	4		16	10	7
			15	5		10	13	2			
		11	8	10	1	6	4	15	3		

Results:

From the start of this activity until the time when students took the posttest totaled three class periods. A total of 76 students took both the pretest and posttest and are therefore included in this analysis. The pretest was given to students prior to any lecture/presentation regarding the specific topics covered therein. The posttest was given the day after all lecture/presentation was completed; the students had not turned in the worksheet (homework) at the time the posttest was given. The posttest was counted as an exam for the students which they could retake if they had not gotten a C or better. We do not include any retake scores in this analysis.

Figure 1 below illustrates the combined results for all 4 classes. As indicated, the average pretest score was 53.62% (4.29 out of 8) and the average posttest score was 60.53% (4.84 out of 8). This was an improvement of 6.91% overall.

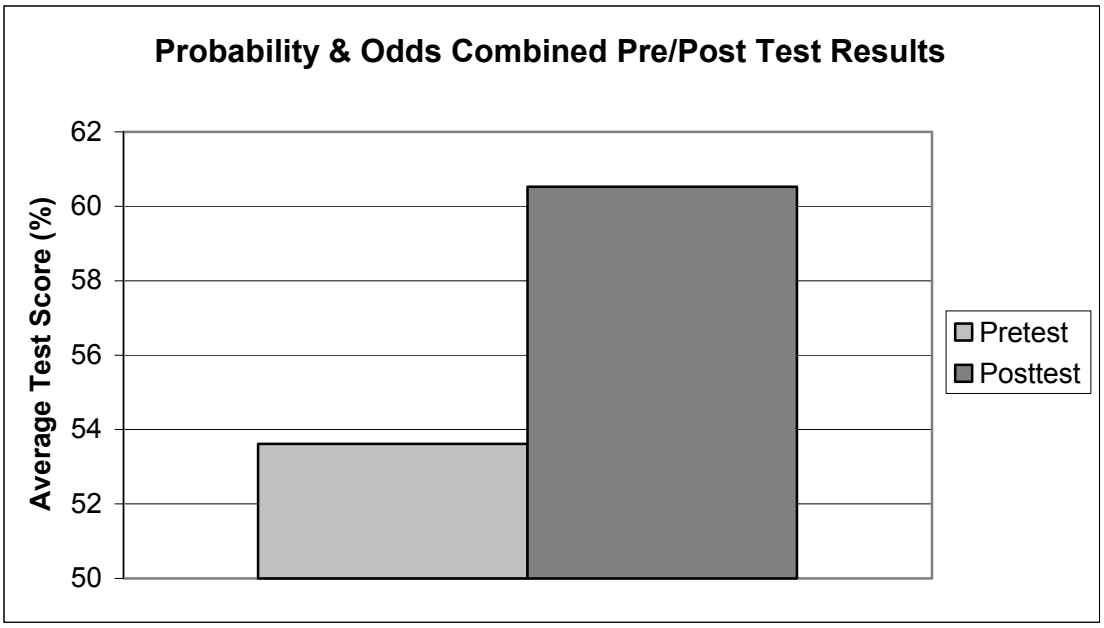

Figure 1

57.89% of the 76 students that took both of the tests showed improvement from the pretest to the posttest. 19.74% of the 76 students had the same score for both the pretest and posttest. 22.37% of the 76 students had a decrease in their score on the posttest.

Table 1 below shows the results for the individual classes as well as the combined outcome. The average pretest score was similar for all periods, ranging from 4.10 to 4.55. The average posttest score was also similar for all periods, ranging from 4.68 to 5.10. The combined results show that the high score for the pretest was six and the high score for the posttest was seven. Furthermore, the lowest score for the pretest was 2 while the low score for the posttest was one. Overall, the percent increase of the average score from the pretest to the posttest was similar for all periods, ranging from 9.75 to 14. 38.

Period	*Average*		*High Score*	*Low Score*	*% Increase of Average Score from Pretest to Posttest*
3	*Pretest*	*4.19*	*6*	*3*	*14.08*
	Posttest	*4.78*	*7*	*2*	
4	*Pretest*	*4.41*	*6*	*2*	*9.75*
	Posttest	*4.84*	*6*	*3*	
5	*Pretest*	*4.10*	*6*	*3*	*14.38*
	Posttest	*4.68*	*6*	*3*	
6	*Pretest*	*4.55*	*6*	*2*	*11.98*
	Posttest	*5.10*	*7*	*1*	
Combined	*Pretest*	*4.29*	*6*	*2*	*12.88*
	Posttest	*4.84*	*7*	*1*	

Table 1

In general, this activity went well in all of the classes. The students really struggled with the first worksheet (homework) that was given; we are unsure if it was lack of motivation, lack of understanding, or some other reason(s) that caused this struggle. To get some sort of assessment on how well students understood the topics covered, we threw out the first worksheet and gave them a new one.