MATH 166
SPRING 2004
EXAM 1

1. (48 pt) Evaluate the following integrals:

a) $\int \frac{e^x}{\sqrt{e^{2x}+6e^x+13}}dx$ b) $\int e^{ax}\cos(bx)dx$ c) $\int \frac{4x^3+4x^2+3x+1}{(x^2+x)(x^2+1)}dx$

d) $\int_1^4 \sqrt{x}e^{\sqrt{x}}dx$ e) $\int \frac{dx}{\sqrt{x}-\sqrt[4]{x}}$ f) $\int_{-R}^{R}(\sin(x)-1)\sqrt{R^2-x^2}\,dx$

2. (12 pt) Suppose that you revolve the upper half of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a,b>0)$ about the $x-$axis. What is the resulting volume? What happens if $a=b$?

3. (18 pt) Consider the curves $\sin(ax)$ and $\cos(ax)$, $a>0$.
   a) Find the area between the curves from $x=0$ to their first point of intersection.
   b) Find the average vertical distance between these curves from $x=0$ to their first point of intersection.
   c) Find the volume obtained when the part of the region bounded by the curves from (from $x=0$ to their first point of intersection) is revolved about the line $x=\frac{\pi}{4a}$.

4. (12 pt) You haul a 10 pound bucket 30 feet up a well. You begin with 100 pounds of water in the bucket, but it is leaking in such a way that you lose 2 pounds of water for every foot that you pull it up. If the rope weighs $\frac{1}{2}$ of a pound per foot, how much work is done in bringing the bucket of water to the top of the well?

5. (10 pt) Let $f(x)$ be a continuous, one to one function. Show that

$$\int_{f(a)}^{f(b)} f^{-1}(x)dx = bf(b)-af(a)-\int_a^b f(u)du.$$

Use this to calculate

$$\int_1^3 g(x)dx$$

where $g(x)$ is the inverse function of $y=x^3+x+1$.

6. (10 pt) Consider a right regular pyramid with a square base. Suppose that the length of one side of the square base of the pyramid is a and height of the pyramid is h. Find the volume of this pyramid.