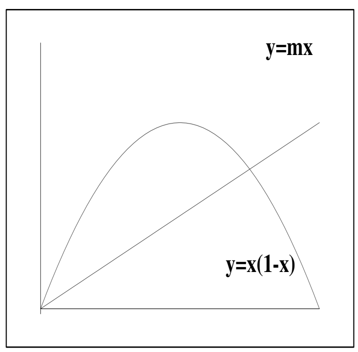
MATH 166
SPRING 2007
EXAM 1

1. (40 pt) Evaluate the following integrals:

$$\text{a)} \int \frac{x^2}{x^2+2x+5}dx \qquad \text{b)} \int x\tan^{-1}(x^2+1)dx \qquad \text{c)} \int \frac{2x^5+4x^3+2x-1}{x^2(x^2+1)}dx$$

$$\text{d)} \int \frac{dx}{4\cos(x)+5} \qquad \text{e)} \int_{2R}^{3R} \sqrt{x^2-2Rx}\, dx,\ R>0$$

2. (24 pt) Consider the curves $f(x) = x - x^2$ and $g(x) = mx$, $0 < m < 1$.
   a) Find the volume obtained when the region bounded by $f(x) = x - x^2$ and the $x-$axis is revolved about the $y-$axis.
   b) Find the area of the region bounded by $f(x)$ and $g(x)$. What happens to the formula you get when $m = 0$ and $m = 1$? Briefly explain.
   c) Find the value of m so that the volume obtained when the region bounded by $f(x)$ and $g(x)$ is revolved about the $y-$axis is exactly half of the volume obtained in part a).



3. (10 pt) Let a be a constant. Find all differentiable functions with the property that $f(x)$ is equal to the average value of $f(x)$ on the interval $[a, x]$ $(x > a)$.

4. (12 pt) A particle moves through a force field where the force on the particle at any point $x > 0$ is given by $f(x) = \frac{x^2-2x+2}{x}$. Find value(s) of a such that the work done as the particle moves from a to $2a$ is minimal.

5. (12 pt) Use integration by parts to find $\int \sec^{2n+1}(x)dx$ in terms of $\int \sec^{2n-1}(x)dx$ (where n is an integer greater than or equal to 1). Apply this formula to find

$$\int \sec^5(x)dx$$

6. (12 pt) A pyramid with square base of length $R > 0$ has its top removed. The new truncated pyramid is h units tall and the side length of the new square top is $r \geq 0$. Find the volume of this solid. What does your formula say when $r = 0$ and when $r = R$? Does this make sense?