MATH 166
SPRING 2009
EXAM 1

1. (50 pt) Evaluate the following integrals:

$$\text{a)} \int (\ln(x))^3\, dx \qquad \text{b)} \int_0^a \frac{x^2}{(x^2+a^2)^{\frac{3}{2}}}\, dx \qquad \text{c)} \int \frac{4e^{4x}+2e^{3x}+45e^{2x}+81}{(e^{2x}+9)^2 e^x}\, dx$$

$$\text{d)} \int \sin(ax)\cos(bx)\, dx \qquad \text{e)} \int_0^1 \tan^{-1}(x) dx$$

2. (12 pt) A bucket weighing a pounds contains b pounds of water. It is hauled up a D foot shaft by a rope that weighs c pounds per foot. Find a formula that tells how much work is done in hauling this bucket of water up the shaft.

3. (6 pt) Let $f(x)$ be a positive, continuous, periodic function of period P (so for all x, we have that $f(x+P) = f(x)$). Let $\mathfrak{R}_1$ be the region bounded by $y = f(x), y = 0, x = 0$, and $x = P$ and let $\mathfrak{R}_2$ be the region bounded by $y = f(x), y = 0, x = P$, and $x = 2P$. Find the volume of the region obtained when $\mathfrak{R}_2$ is revolved about the line $x = -P$ in terms of V and A where A is the area of the region $\mathfrak{R}_1$ and V is the volume obtained when $\mathfrak{R}_1$ is revolved about the $y-$axis.

4. Consider the ellipse $\frac{(x-R)^2}{a^2} + \frac{y^2}{b^2} = 1$ which is "centered" at $(R, 0)$ where $R \geq a$.
   a) (8 pt) Find the volume obtained when the upper half is revolved about the $x-$axis.
   b) (8 pt) Find the volume obtained when this ellipse is revolved about the $y-$axis.
   c) (5 pt) What relationship between b and R is required for the answers in parts a) and b) to be the same?

5. (15 pt) Suppose that a conical shaped pool is built into the ground of radius R and height (depth) h. Show that the amount of work done in pumping all of the water out of the pool is given by $W = \frac{1}{4}Fh$ where F is the weight of the water in the pool. The volume of a cone is given by $V = \frac{1}{3}\pi R^2 h$ where R is the base radius and h is the height.

6. (6 pt) Imitate the slicing procedure for finding volumes to obtain the volume of a four dimensional sphere of radius R (hint: set up an axis and instead of circles being at every point, assume it is spheres...the volume of a 3 dimensional sphere of radius r is given by $V = \frac{4}{3}\pi r^3$).