MATH 166
SPRING 2011
EXAM 1

1. (50 pt) Evaluate the following integrals:

a) $\int \frac{\ln(\tan^{-1}(x))}{x^2+1}\,dx$ b) $\int \frac{e^{3x}}{\sqrt{e^{2x}-1}}\,dx$ c) $\int_{-2}^{0} \sqrt{5-8x-2x^2}\,dx$

d) $\int \frac{d\theta}{\csc(\theta)-1}$ e) $\int_0^{\frac{1}{a}} x\sin^{-1}(ax)dx, a>0$

2. (15 pt) Find the volume obtained when the right half of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a,b>0)$ is revolved about the $y-$axis. What happens if $a=b$?

3. (15 pt) Consider the region in the first quadrant bounded by the upper half of the circle $x^2+y^2=R^2$, the line $y=(\tan(\theta))x$ and the $x-$axis. Find the volume obtained when this region is revolved about the $x-$axis. What happens as $\theta \longrightarrow 0$ and what happens as $\theta \longrightarrow \frac{\pi}{2}$?

4. (12 pt) A monument is to be constructed in the shape of a truncated pyramid. The cross sections parallel to the ground are squares and the base square has side length b and the top has side length a $(b \geq a \geq 0)$. The pyramid has height $h>0$ and all lengths are measured in feet. If the concrete used to make the monument weighs ρ pounds per cubic foot, find the work done in building this monument.

5. (10 pt) Consider the triangle bounded by the $x-$axis, the $y-$axis, and the line $y=1-x$. Find the volume obtained when this region is revolved about the line $y=-x$.

6. (8 pt) Let $f(x)$ be a continuous, periodic function of period $T>0$ (this means that $f(x+T)=f(x)$ for all x).

a) Show that for all real numbers a, we have that $\int_a^{a+T} f(x)dx = \int_0^T f(x)dx$.
b) Show that if $f(x)$ is additionally assumed to be odd, then the average value of $f(x)$ on $[a, a+T]$ is 0 for all real numbers a.