MATH 166
SPRING 2012
EXAM 1

1. (50 pt) Evaluate the following integrals:

a) $\int \sin(x)\cos(x)\ln(\sin(x))\,dx$ b) $\int \frac{\sqrt{1-9x^2}}{x}\,dx$ c) $\int_0^4 \sqrt{4x-x^2}\,dx$

d) $\int_1^2 \frac{2x^3+2x^2+1}{x^4+x^2}\,dx$ e) $\int e^{2x}\sin(3x)dx$

2. (15 pt) An object has a base shaped like a circle. Cross sections perpendicular to the base and a particular diameter are all in the shape of rectangles that are twice as tall as they are wide. If the radius of the circle is R, find the volume of this object.

3. (15 pt) Imagine that you have a large sphere of radius R. You take a large cylindrical soup can of radius $r \leq R$ (and very long length) and drill through the sphere (with the central axis of the can coinciding with the north-south pole diameter of the sphere). Find the volume of the part of the sphere that is inside the cylindrical drill.

4. (12 pt) A spherical storage tank of radius R is buried L units below the surface of the ground. If the sphere is filled with a liquid of density ρ, how much work is required to pump the tank half empty.

5. (10 pt) Let $n \geq 1$. Find the area between the curves $y = x^n$ and $x = y^n$, $x > 0$. What happens to your answer if $n = 1$? What happens as $n \longrightarrow \infty$? Does this make sense?

6. Let $f(x)$ be a function that is continuous everywhere and $\alpha > 0$ a fixed number. We consider all intervals $[t, t+\alpha]$ of length α. Let $A(t)$ be the average value of $f(x)$ on $[t, t+\alpha]$.

a) (6 pt) Show that if $A(t)$ has a maximum or a minimum, then $f(x)$ has the same value on the endpoints of the interval (that is, $f(t) = f(t+\alpha)$).

b) (2 pt) Show that if $f(x) = x^3 - 3x$, and $\alpha > \sqrt{12}$ then $A(t)$ has neither a maximum nor a minimum.