

MATH 166
SPRING 2005
EXAM 2

1. (32 pt) Evaluate the following integrals if they exist:

a) $\int_0^{\frac{\pi}{2}} \frac{\cos(x)dx}{\sqrt{\sin(x)}}$ b) $\int_{-\infty}^{\infty} \frac{e^{-\frac{1}{|x|}}}{x^2} dx$ c) $\int_0^1 \frac{e^x dx}{\sqrt{e^x - 1}}$ d) $\int_{e^e}^{\infty} \frac{dt}{t \ln(t) \ln(\ln(t))}$

2. Consider the functions $f(x) = \ln |\sec(x)|$ and $g(x) = \sin(x)$.

a) (10 pt) Find the arclength of the function $f(x)$ from $(0, 0)$ to $(\frac{\pi}{4}, \ln(\sqrt{2}))$.

b) (5 pt) Find the value of a , $(0 \leq a \leq \pi)$ such that the length of $g(x)$ from $(a, \sin(a))$ to $(a + \frac{\pi}{2}, \sin(a + \frac{\pi}{2}))$ is maximized.

3. (12 pt) Consider the circle given by the equation $x^2 + (y - R)^2 = r^2$ with $r \leq R$. Find the surface area obtained when the region defined by this circle is revolved about the x -axis.

4. (8 pt) Show that the integral

$$\int_0^{\infty} \frac{\sin(e^x) + 1}{x^2 + 1} dx$$

converges.

5. Consider the quarter of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ that is in the first quadrant.

a) (12 pt) Find the centroid of this region.

b) (4 pt) Find the volume obtained when this region is revolved about the x -axis.

c) (4 pt) Find the volume obtained when this region is revolved about the y -axis.

6. (8 pt) Consider the integral

$$\int_0^2 (4x^3 - 3x^2 + 2x - 6) dx.$$

Find the approximation S_{14} (Simpson's rule) for this integral. Estimate the error!

7. (15 pt) A semicircular window is installed in an aquarium. The window is placed with the straight side at the top and the semicircular side below. Suppose that the top of the window is at depth d below the water and the radius of the semicircle is R (both in feet). Find the force due to hydrostatic pressure on this window. (You may assume that the pressure of the water in the aquarium is given by 62.5 times the depth).