

MATH 166
SPRING 2008
EXAM 2

1. (32 pt) Evaluate the following integrals if they exist:

a) $\int_0^{\frac{\pi}{2}} \sin(x)\sqrt{\sec(x)}dx$ b) $\int_1^{\infty} \left(\frac{1}{x} - \frac{1}{\sqrt{x^2+1}}\right)dx$ c) $\int_{-\infty}^{\infty} |x|e^{-|x|}dx$ d) $\int_1^{\infty} \sin\left(\frac{2}{x}\right)dx$

2. (15 pt) Show that if

$$\int_a^{\infty} f(x)dx$$

converges, and $\lim_{x \rightarrow \infty} f(x)$ exists, then $\lim_{x \rightarrow \infty} f(x) = 0$. Is the converse true? That is, if $f(x)$ is a function such that $\lim_{x \rightarrow \infty} f(x) = 0$, does this mean that $\int_a^{\infty} f(x)dx$ exists for some a ?

3. (18 pt) Consider the function $f(x) = \ln(x)$, and let $0 < t < 1$.

- a) Find the length of this curve on the interval $[t, 1]$. What happens to your answer as $t \rightarrow 0^+$?
- b) Find the surface area obtained when this curve on $[t, 1]$ is revolved about the y -axis. What happens to your answer as $t \rightarrow 0^+$?

4. Consider the region \mathfrak{R}_1 to be the region inside the circle $x^2 + y^2 = R^2$ and bounded by the angles θ and $-\theta$ where $\frac{\pi}{2} \leq \theta \leq \pi$.

- a) (5 pt) Find the area of this region.
- b) (10 pt) Locate the centroid of this region. What happens to your answer when $\theta = \pi$?

5. (15 pt) A glass sphere of radius R is filled with a fluid of density ρ . Find the proportion of the force due to hydrostatic pressure on the southern hemisphere to the force due to hydrostatic pressure on the northern hemisphere.

6. (15 pt) Consider the function

$$f(x) = \begin{cases} b, & \text{if } 0 \leq x < a; \\ c, & \text{if } x = a; \\ d, & \text{if } a < x \leq 2a. \end{cases}$$

- a) Evaluate $\int_0^{2a} f(x)dx$.
- b) For what values of c is $M_2 = \int_0^{2a} f(x)dx$?
- c) For what values of c is $S_2 = \int_0^{2a} f(x)dx$?