MATH 166
SPRING 2009
EXAM 2

1. (40 pt) Evaluate the following integrals if they exist:

a) $\displaystyle\int_0^{\infty} (x - \sqrt{x^2+1})dx$ b) $\displaystyle\int_{-\infty}^{\infty} \frac{e^{-\sqrt[3]{|x|}}}{x^{\frac{2}{3}}}\, dx$ c) $\displaystyle\int_{-R}^{R} \frac{dx}{\sqrt{R^2-x^2}}$ d) $\displaystyle\int_{-e}^{e^2} \frac{(\ln(|t|))^n dt}{t}, n > 0$

2. (20 pt) Let $a > 0$ be a fixed constant. Find all values of b ($a \geq b \geq 0$) such that the centroid of the region bounded by $f(x) = ax(1-x)$ and $g(x) = bx(1-x)$, $0 \leq x \leq 1$ is actually in the region.

3. (8 pt) Show that the substitution $x = \tan(\theta)$ converts the integral

$$\int_1^{\infty} \frac{\sqrt{x^2+1}}{x^n} dx, n \text{ an integer greater than } 2$$

to a proper integral (and hence the integral converges).

4. (15 pt) Find the surface area obtained when $f(x) = \sin(nx)$, where n is a positive integer and $0 \leq x \leq \frac{\pi}{n}$ is revolved about the $x-$axis. What happens to your answer as $n \longrightarrow \infty$ (does this make sense)?

5. (8 pt) Find a such that the length of the curve $f(x) = x^3$ on the interval $[a, a+k]$ (where $k > 0$ is a fixed constant) starting at a is minimized.

6. (12 pt) Find the force due to hydrostatic pressure on the side of a trough of length L if the end is a semicircle of radius R and it is filled to the top with a fluid of density ρ.

7. (7 pt) Explain the construction behind Simpson's Rule and why the value of n must always be even.