MATH 166
SPRING 2011
EXAM 2

1. (40 pt) Evaluate the following integrals if they exist:

a) $\displaystyle\int_1^\infty \frac{dx}{x\sqrt{x-1}}$ b) $\displaystyle\int_{-\infty}^{\infty} xe^{-x}\,dx$ c) $\displaystyle\int_1^\infty \frac{dx}{(\ln(x))^2+1}$ d) $\displaystyle\int_0^{\frac{\pi}{2}} (\sec(x)-\tan(x))dx$

2. (15 pt) Consider the function $f(x) = Ax(1-x), 0 \le x \le 1$. Find the surface area generated when this function is revolved about the $y-$axis (you may freely use the results from problem 5). What happens to your answer as $A \longrightarrow 0^+$? Does this make sense?

3. (15 pt) Let $g(x) = \int_a^x \sqrt{(f'(t))^2-1}\,dt$ (we assume $f'(t) \ge 1$ for all t). Find the length of $g(x), a \le x \le b$.

4. (10 pt) Suppose that $f(x)$ is periodic of period P (that is, $f(x+P) = f(x)$ for all x) and we wish to estimate $\int_0^{2P} f(x)dx$ using Simpson's rule.

a) Find the error bound for Simpson's rule for $\int_0^P f(x)dx$ using n intervals and the error bound for $\int_0^{2P} f(x)dx$ using $2n$ intervals and compute the ratio of the first error bound to the second one.
b) Which is a better bet for accuracy: using Simpson's rule to estimate $\int_0^{2P} f(x)dx$ with $2n$ intervals or using Simpson's rule for $\int_0^P f(x)dx$ using n intervals and doubling the result? Explain.

5. (15 pt) Suppose that $f(x), 0 \le x \le 2a$ is symmetric about the line $x = a$ (that is $f(x) = f(2a-x)$).
a) Show that $f'(x) = -f'(2a-x)$.
b) Show that $\int_a^{2a} 2\pi x\sqrt{1+(f'(x))^2}dx = \int_0^a 2\pi(2a-u)\sqrt{1+(f'(u))^2}du$.
c) Show that if S is the surface area of the surface obtained when $y = f(x), 0 \le x \le 2a$ is revolved about the $y-$axis, then $S = 4\pi aL$ where L is the length of $y = f(x), 0 \le x \le a$.

6. (15 pt) Consider a conical tank of height h and radius R (with the vertex pointing toward the ground). Find the force due to hydrostatic pressure on the cone. Then show that the total force divided by the surface area of the side of the cone depends only on the height of the cone.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) Force=(pressure)(area) and pressure=ρ(depth).
(12) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(13) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(14) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(15) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(16) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(17) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(18) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(19) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$
(20) The surface area of a cone: $A = \pi r L$ where r is the radius and L is the slant height.