MATH 166
SPRING 2012
EXAM 2

1. (60 pt) Evaluate the following integrals if they exist:

a) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2(x)dx}{\sqrt{|\tan(x)|}}$ b) $\displaystyle\int_0^{\infty} \frac{\sin(\tan^{-1}(x))}{1+x^2}\,dx$ c) $\displaystyle\int_0^{\infty} \frac{x\sin(x)+2x}{x^2+1}\,dx$ d) $\displaystyle\int_0^{\frac{\pi}{2}} (\csc(x) - \frac{1}{x})dx$

2. (10 pt) A porthole on an ocean liner is in the shape of a circle of radius R and the top of the window is D units below the surface of the water. If the window can only withstand 5 times the force on it when it is in its "natural" position, how deep can the top of the window go underwater before it implodes.

3. (10 pt) Show that there is no differentiable function $f(x)$ such that the length of the function from $x = 0$ to $x = a$ $(0 < a \leq \frac{\pi}{2})$ is equal to $\sin(a)$.

4. (10 pt) Let $f(x)$ be a differentiable function and let S be the surface area obtained when $f(x)$, $0 \leq x \leq a$ is revolved about the $y-$axis. Find the surface area obtained when $f(x)$, $0 \leq x \leq a$ is revolved about the line $x = -b^2$ in terms of S, b^2, and L, the length of the curve from 0 to a.

5. (8 pt) Suppose that you wish to use Simpson's rule to evaluate

$$\int_{\frac{1}{2}}^{\frac{3}{2}} F(x)dx$$

where $F'(x) = \int x(\ln(x))^2 dx$. Find the smallest value of n such that the error obtained in the approximation $S_n \approx \int_{\frac{1}{2}}^{\frac{3}{2}} F(x)dx$ has error strictly less than $\frac{1}{900000}$.

6. (6 pt) Find the sixth degree Taylor polynomial of $f(x) = \sin(x)$ centered at $x = 0$.

7. (6 pt) Find the volume of the torus obtained when the circle $x^2 + (y - R)^2 = r^2$, $0 < r < R$ is revolved about the $x-$axis.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2} dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$
(12) $S = \int_a^b 2\pi(x \text{ or } y) ds$
(13) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(14) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(15) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(16) $A = \int_a^b \frac{1}{2} r^2 d\theta$
(17) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(18) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$
(19) $\int \csc(x)dx = \ln|\csc(x) - cot(x)| + c$
(20) $\sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$