MATH 166
SPRING 2013
EXAM 2

1. (40 pt) Evaluate the following integrals if they exist:

a) $\displaystyle\int_0^{\infty} \frac{\cos(x)}{\sin^2(x)+1}dx$ b) $\displaystyle\int_0^{9} \frac{1}{\sqrt[3]{x^2-2x+1}}dx$ c) $\displaystyle\int_1^{\infty} \frac{\ln(x)}{x^p}dx,\ p>1$ d) $\displaystyle\int_0^{\infty} \frac{a^2}{(x^2+a^2)^{\frac{3}{2}}}dx$

2. (20 pt) Consider the function $f(x) = a^2x(\frac{1}{a} - x)$.

a) Find the length of this curve $0 \leq x \leq \frac{1}{a}$.
b) What happens as $a \longrightarrow \infty$?
c) What is the surface area obtained when this curve ($0 \leq x \leq \frac{1}{a}$) is revolved about the $x-$axis?
d) What happens as $a \longrightarrow \infty$?

3. (10 pt) Suppose you have a function $F(x)$ and you want to use Simpson's rule to estimate $\int_0^4 F(x)dx$. You know that $F''(x) = \int_0^x \ln(t^2+1)dt$. How many terms do you need if you want to guarantee that the error obtained in approximating $\int_0^4 F(x)dx$ is strictly less than $\frac{1}{720}$?

4. (10 pt) In this problem, we ultimately want to find $\lim_{a\to\infty} \int_0^{\frac{1}{a}} \sqrt{1+a^2e^{2ax}}dx$.

a) Find a function and an interval so that $\int_0^{\frac{1}{a}} \sqrt{1+a^2e^{2ax}}dx$ represents the length of this function on that interval.
b) Use part a) (or any other method) to evaluate $\lim_{a\to\infty} \int_0^{\frac{1}{a}} \sqrt{1+a^2e^{2ax}}dx$.

5. (10 pt) A tank is made by revolving the right half of parabola $y = x^2,\ 0 \leq x \leq \sqrt{h}$ about the $y-$axis. If this tank is filled with a fluid of density ρ, find the force due to hydrostatic pressure on the tank.

6. (20 pt) Consider the region bounded by the $x-$axis and the function $f(x) = \sin(x),\ 0 \leq x \leq \pi$.

a) Locate $\overline{x}$.
b) Locate $\overline{y}$.
c) Find the volume obtained when this region is revolved about the $x-$axis.
d) Find the volume obtained when this region is revolved about the $y-$axis.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(12) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(13) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(14) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(15) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(16) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(17) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(18) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$
(19) $\int \sec^5(x)dx = \frac{1}{4}\sec^3(x)\tan(x) + \frac{3}{8}\sec(x)\tan(x) + \frac{3}{8}\ln(|\sec(x) + \tan(x)|) + c$
(20) $\int \csc(x)dx = \ln|\csc(x) - \cot(x)| + c$
(21) $\sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$