

MATH 166
SUMMER 2011
EXAM 2

1. (60 pt) Evaluate the following integrals.

a) $\int \frac{x^2}{(x^2 + 9)^{\frac{3}{2}}} dx$ b) $\int_1^2 \sqrt{3 + 2x - x^2} dx$ c) $\int \frac{4x^4 + 2x^3 + 5x^2 + 1}{x^2(x^2 + 1)^2} dx$
d) $\int 4x \sin(x) \cos(x) dx$ e) $\int \cos(\ln(x)) dx$

2. (20 pt) Evaluate the following integrals.

a) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)^2} dx, a > 0$ b) $\int_0^{\infty} \frac{dx}{\sqrt{x}(x^2 + 4x + 3)}$

3. (20 pt) Consider the function $f(x) = \int_0^x e^{3t^2 - t^3} dt$ and let $g(x) = 4x + 1 + F(x)$ where $F(x)$ is an antiderivative of $f(x)$. Suppose that we want to estimate $\int_0^3 g(x) dx$ using the trapezoid rule.

- a) Find the relevant value of K for estimating the error in using the trapezoid rule to estimate $\int_0^3 g(x) dx$.
- b) If we want the error in using T_n to be less than $\frac{1}{400}$, find the smallest appropriate value of n such that this error bound is achieved.

4. (10 pt) Consider the region bounded by the function $f(x) = \frac{1}{x^n}, (n > 1), x \geq 1$ and the x -axis. Find the volume obtained when this region is revolved about the x -axis.

Formulae

- (1) $\sin(2x) = 2 \sin(x) \cos(x)$
- (2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
- (3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x)$
- (4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x)$
- (5) $\sin(A) \cos(B) = \frac{1}{2} [\sin(A - B) + \sin(A + B)]$
- (6) $\sin(A) \sin(B) = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$
- (7) $\cos(A) \cos(B) = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$
- (8) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
- (9) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
- (10) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
- (11) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
- (12) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
- (13) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
- (14) $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$
- (15) $S = \int_a^b 2\pi(x \text{ or } y) ds$
- (16) $\int_{n+1}^{\infty} f(x) dx \leq R_n \leq \int_n^{\infty} f(x) dx$
- (17) $\bar{x} = \frac{1}{A} \int_a^b x(f(x) - g(x)) dx$
- (18) $\bar{y} = \frac{1}{2A} \int_a^b [(f(x))^2 - (g(x))^2] dx$
- (19) $A = \int_a^b \frac{1}{2} r^2 d\theta$
- (20) $\int \sec(x) dx = \ln |\sec(x) + \tan(x)| + c$
- (21) $\int \sec^3(x) dx = \frac{1}{2} \sec(x) \tan(x) + \frac{1}{2} \ln |\sec(x) + \tan(x)| + c$
- (22) If $t = \tan\left(\frac{x}{2}\right)$ then $\sin(x) = \frac{2t}{t^2+1}$, $\cos(x) = \frac{1-t^2}{t^2+1}$, $dx = \frac{2dt}{t^2+1}$