MATH 166
SPRING 2004
EXAM 3

1. (42 pt) Determine if the following series converge or diverge.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{2n^3+n}}{\sqrt{n^5+\sin(n)}}$ b) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ c) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$ d) $\displaystyle\sum_{n=1}^{\infty} \frac{(4n)!}{17^n((2n)!)^2}$

e) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\tan^{-1}(n)}{n+1}$ f) $\displaystyle\sum_{n=1}^{\infty} a_n$, where the partial sums are given by $s_n = n\sin\left(\frac{3}{2n}\right)$.

2. (12 pt) Determine if the following sequences converge or diverge.

a) $\left\{an\tan\left(\frac{b}{cn}\right)\right\}_{n=1}^{\infty}$, $c \neq 0$ b) $\left\{e^{-s_n}\right\}_{n=1}^{\infty}$, where s_n is the n^{th} partial sum of a positive term series.

3. (12 pt) Suppose that $\sum_{n=0}^{\infty} a_n$ is a positive term series.
   a) Show that if $\sum_{n=0}^{\infty} a_n$ converges then so does $\sum_{n=0}^{\infty} a_n^2$.
   b) Show that if $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} \sin(a_n)$ both converge or both diverge.

4. (16 pt) Consider the two convergent series $\sum_{n=2}^{\infty}(-1)^n \frac{1}{n(\ln(n))^2}$ and $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$.
   a) Show that 18 terms is (more than) enough so the the approximation $s \approx s_n$ for the series $\sum_{n=2}^{\infty}(-1)^n \frac{1}{n(\ln(n))^2}$ has error less than or equal to $\frac{1}{100}$ (hint: $\ln(20) > \sqrt{5}$).
   b) How many terms are needed so that the approximation $s \approx s_n$ for the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$ has error less than or equal to $\frac{1}{100}$?

5. (12 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=1}^{\infty}(-1)^n \frac{\ln(n)(x+1)^{3n}}{(n+1)27^n}.$$

6. (8 pt) Find a Maclaurin series for $f(x) = \tan^{-1}(2x^2)$ and use this series to estimate $\int_0^{\frac{1}{2}} \tan^{-1}(2x^2)dx$ with error less than or equal to $\frac{1}{1000}$.

7. (8 pt) Suppose that the power series $\sum_{n=0}^{\infty} a_n x^n$ converges on the interval $(-R, R)$ (that is, the series has radius of convergence $R > 0$).
   a) What is the radius of convergence of the series $\sum_{n=0}^{\infty} a_n(Ax^k)^n$ (where k is a positive integer and A is nonzero)?
   b) For what values of x does the infinite series $\sum_{n=0}^{\infty} a_n(\frac{1}{x})^n$ converge?