MATH 166
SPRING 2005
EXAM 3

1. (42 pt) Determine if the following series converge or diverge.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{3n^5+2n^3+n^2+12}}$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$ c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(n!)^2}{(2n)!}$ d) $\displaystyle\sum_{n=1}^{\infty} (\sqrt[n]{3}-\sqrt[n]{2})^{2n}$

e) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n\ln(n)}{n^2+1}$ f) $\displaystyle\sum_{n=1}^{\infty} a_n$, where the partial sums are given by $s_n = \dfrac{n+2}{2n}$.

2. (10 pt) Determine if the following sequences converge or diverge.

a) $\left\{\dfrac{n^2+n\tan^{-1}(n)}{n^2+1}\right\}_{n=1}^{\infty}$

b) $\Big\{|\sin(x)|, \sin(|\sin(x)|), \sin(\sin(|\sin(x)|)), \cdots\Big\}$ (Hint: if $0 \le \theta \le \dfrac{\pi}{2}$ then $0 \le \sin(\theta) \le \theta$.)

3. (15 pt) Consider the function

$$f(x) = \begin{cases} 2x-\pi+1, \text{if } x \ge \frac{\pi}{2} \\ \sin(x), \text{if } x < \frac{\pi}{2}. \end{cases}$$

a) Find the Maclaurin series for $f(x)$ and determine its radius of convergence.
b) Find the Taylor series centered for $f(x)$ centered at π and determine its radius of convergence.
c) Is $f(x)$ equal to either of the series above for all x? Explain.

4. Let $f(x)$ be a function that is equal to its Taylor series (with an infinite radius of convergence) centered at any point a.

a) (8 pt) Show that

$$f(0) = \sum_{n=0}^{\infty} (-1)^n \frac{f^{(n)}(a)}{n!} a^n.$$

b) (5 pt) Use the above formula with $f(x) = \cos(x)$ to show that $1 = \cos^2(a) + \sin^2(a)$ (hint: write your series as two series consisting of the even and odd powers of a and regroup).

5. (12 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{\sqrt{(n+1)}(x+1)^{2n}}{9^n n^3}.$$

6. (10 pt) Consider the function $f(x) = \sin(\frac{1}{x^2})$.

a) Use the MacLaurin series for $\sin(x)$ to find a series for $f(x)$.
b) Use your answer from a) to estimate $\int_1^{\infty} \sin(\frac{1}{x^2})dx$ with error less than or equal to $\frac{1}{500}$.

7. (8 pt) Consider the p–series $\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$. We will assume that $p > 1$ (so the series converges). Note that no matter what p is, the first term of the series is 1. What is the smallest value of p such that the estimate $s \approx 1$ has error less than or equal to $\frac{1}{1000}$.