MATH 166
SPRING 2006
EXAM 3

1. (36 pt) Determine if the following series converge or diverge.

$$\text{a)} \sum_{n=1}^{\infty} \frac{n^3}{\sqrt{n^7+1}} \qquad \text{b)} \sum_{n=1}^{\infty} n^2 \sin(\frac{1}{n^4}) \qquad \text{c)} \sum_{n=0}^{\infty} \frac{28^n (n!)^3}{(3n)!} \qquad \text{d)} \sum_{n=1}^{\infty} \frac{(2n^2+1)^{3n}}{(3n^3-1)^{2n}}$$

$$\text{e)} \sum_{n=2}^{\infty} (-1)^n \frac{\ln(\ln(n))}{\ln(n)} \qquad \text{f)} \sum_{n=1}^{\infty} \sin(a_n), \text{ where } \sum_{n=1}^{\infty} a_n \text{ is a convergent, positive term series.}$$

2. (12 pt) Determine if the following sequences converge or diverge.

$$\text{a)} \Big\{ n \tan(\frac{1}{n}) \Big\}_{n=1}^{\infty}$$

$$\text{b)} \Big\{ a_n \Big\}_{n=1}^{\infty} \text{ where } a_1 = 1 \text{ and } a_{n+1} = 3 - \frac{1}{a_n} \text{ for } n \geq 1.$$

3. (15 pt) Suppose that the power series $\sum_{n=0}^{\infty} a_n x^n$ has interval of convergence $[-R, R]$ $(R > 0)$ and that the series converges absolutely at the endpoints. Find the interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{a_n}{n+1} (ax-b)^{cn}$$

where $a \neq 0$ and c is a positive integer.

4. Let $R > 0$ and suppose that $\sum_{n=0}^{\infty} a_n x^n$ converges if $|x| < R$ and diverges if $|x| > R$.
   a) (10 pt) Show that $\sum_{n=0}^{\infty} a_n (\frac{1}{x})^n$ converges for $|x| > \frac{1}{R}$ and diverges for $|x| < \frac{1}{R}$.
   b) (5 pt) Suppose that $f(x)$ is equal to its Maclaurin series and has the property that $f(x) = f(\frac{1}{x})$ for all $x \neq 0$. Explain why if $R > 1$, then $f(x)$ is continuous everywhere and if $R = 1$ then $f(x)$ has at most two discontinuities. What happens if $R < 1$?

5. (12 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{(2n+1)}(3x-6)^{2n}}{9^n}.$$

6. (10 pt) Evaluate the following limit (you might try using a series).

$$\lim_{x \to 0} \frac{\sin(x^2) + e^{-x^2} - \cos(x^3)}{3x^4}$$

7. (10 pt) Estimate

$$\int_0^1 \frac{dx}{16+x^4}$$

with error less than or equal to $\frac{1}{20000}$ (and explain how you come to your conclusions).