MATH 166
SPRING 2007
EXAM 3

1. (36 pt) Determine if the following series converge or diverge.

$$\text{a)} \sum_{n=1}^{\infty} \frac{\sqrt{10n^3+5}}{\sqrt[6]{2n^{16}+n^2+1}} \qquad \text{b)} \sum_{n=1}^{\infty} \ln(n)\sin\left(\frac{1}{n}\right) \qquad \text{c)} \sum_{n=1}^{\infty} (a^{\frac{1}{n}}-1), a>1$$

$$\text{d)} \sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} \qquad \text{e)} \sum_{n=2}^{\infty} (-1)^n \frac{\ln(n^{100})}{n}$$

f) $\sum_{n=1}^{\infty} f(a_n)$, where $\sum_{n=1}^{\infty} a_n$ is a convergent, positive term series and $f(x)$ is a positive, function with $f'(x)$ continuous and $f(0)=0$.

2. (12 pt) Determine if the following sequences converge or diverge.

a) $\left\{s_n - \ln(n)\right\}_{n=1}^{\infty}$ where s_n is the n^{th} partial sum of $\sum_{k=1}^{\infty} \frac{1}{k}$.

b) $\left\{an^k \sin\left(\frac{b}{n^k}\right)\right\}_{n=1}^{\infty}$, $a, b \neq 0, k > 0$

3. Consider the infinite series

$$\sum_{p=2}^{\infty}\sum_{n=2}^{\infty} \frac{1}{n^p}$$

a) (5 pt) Explain why the terms in this sum can be rearranged at will.
b) (10 pt) Show that this series converges and find the sum.

4. (10 pt) Suppose you want to obtain a Taylor series for

$$f(x) = \sqrt{|x^2-5x+1|}$$

centered at $c=1$. If $f(x)$ is equal to its Taylor series centered at $c=1$, what is the maximum value that the radius of convergence of your series could be?

5. (15 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n}(3x-2)^n.$$

6. (12 pt) Find the Taylor series for $f(x)=\ln(x)$ centered at $c=1$. Use this series to estimate $\ln(\frac{3}{2})$ with error less than $\frac{1}{100}$.

7. Estimations.
a) (5 pt) Estimate $\int_0^{\frac{1}{2}} \cos(x^3)dx$ with error less than or equal to $\frac{1}{10000}$ (and explain how you come to your conclusions).
b) (5 pt) What is the smallest value of n for which the estimate $s \approx s_n$ for the convergent series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ has error less than or equal $\frac{1}{100}$.