MATH 166
SPRING 2008
EXAM 3

1. (42 pt) Determine if the following series converge of diverge.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n!(3n)!}{((2n)!)^2}$ b) $\displaystyle\sum_{n=1}^{\infty}(e^{\frac{1}{n^2}} - 1)$ c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n(1 - n\sin(\frac{1}{n}))$ d) $\displaystyle\sum_{n=2}^{\infty} \frac{\sin(e^n)\cos(e^{n!})}{n^2\ln(n)}$

e) $\displaystyle\sum_{n=3}^{\infty} \frac{\sqrt[a]{n^k - 1}}{\sqrt[b]{n^j - 1}}$, where $\dfrac{j}{b} - \dfrac{k}{a} > 1$. f) $\displaystyle\sum_{n=2}^{\infty} \frac{(2\tan^{-1}(n!))^{2n}}{9^n}$

2. (14 pt) Determine if the following sequences converge or diverge.

$$\text{a)}\left\{(-1)^n(1 + \frac{1}{n})^n\right\}_{n=1}^{\infty} \qquad \text{b)}\left\{a_n\right\}_{n=1}^{\infty}, \text{where } a_1 = 1 \text{ and } a_{n+1} = \frac{1}{3 - a_n}.$$

3. (16 pt) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ and $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^p}$, with $p > 1$ and let ϵ be a positive number. Suppose you want to know how many terms are necessary to estimate these series with error less than or equal to ϵ.

a) Find a formula in terms of ϵ and p that indicates the number of terms necessary to approximate $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^p}$ with error less than or equal to ϵ.

b) Find a formula in terms of ϵ and p that indicates the number of terms necessary to approximate $\sum_{n=1}^{\infty} \frac{1}{n^p}$ with error less than or equal to ϵ.

4. (12 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{\ln(n+1)}{n2^{3n}}(4x+2)^{n+1}.$$

5. (10 pt) Consider the function $f(x) = e^{-x^2}$ and let $F(x)$ be an antiderivative of $f(x)$ such that $F(0) = 1$. Use a Maclaurin series to find

$$\int_0^{\frac{1}{2}} F(x)dx$$

with error less than $\frac{1}{2000}$.

6. (8 pt) Suppose that $\sum_{n=0}^{\infty} a_n$ is a positive term series. Show that if it can be successfully limit compared to a $p-$series (in the sense that the limit is a number L such that $0 < L < \infty$) then the ratio test will always be inconclusive for the series.

7. (8 pt) Evaluate

$$\lim_{x\to 0} \frac{e^{-2x^2} + 2e^{x^2} - 3\cos(3x^2)}{x^4}.$$