MATH 166
SPRING 2010
EXAM 3

1. (48 pt) Determine if the following series converge of diverge.

a) $\sum_{n=2}^{\infty} \frac{1}{\ln(n^n)}$ b) $\sum_{n=1}^{\infty} (\sqrt[n]{5} - (\frac{1}{2})3^{-\frac{1}{n}})^{2n}$ c) $\sum_{n=1}^{\infty} \frac{n!(2n)!6^n}{(3n)!}$ d) $\sum_{n=2}^{\infty} (-1)^n (\frac{n+\ln(n)}{2n})$

e) $\sum_{n=3} \frac{\cos(e^{n!})}{n^2+1}$ f) $\sum_{n=2}^{\infty} \frac{\sqrt{n^5+n+1}}{\sqrt[4]{2n^{18}+n^7+4n+2}}$

2. (16 pt) Determine if the following sequences converge or diverge.

a) $\left\{ \sqrt{3}, \sqrt{1+\sqrt{3}}, \sqrt{1+\sqrt{1+\sqrt{3}}}, \cdots \right\}_{n=1}^{\infty}$

b) $\left\{ \frac{f(n)}{2n+1} \right\}_{n=1}^{\infty}$, where, for all $n \geq 0$, $-4\tan^{-1}(n) \leq f(n) \leq 2\tan^{-1}(n)$.

3. (12 pt) Consider the series $\sum_{n=0}^{\infty} \frac{1}{n!}$.

a) Show that this series converges (any way that you like).
b) What is the sum of this series?
c) Show that $(\sum_{n=0}^{\infty} \frac{1}{n!})^3 = \sum_{n=0}^{\infty} \frac{3^n}{n!}$.

4. (12 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n}{(n^2+1)5^n}(2x-1)^{2n}.$$

5. (12 pt) Consider the function $f(x) = \frac{1}{1+x^4}$. We wish to use a series to estimate $\int_0^a f(x)dx$.

a) For what values of $a > 0$ can we use a series to estimate this integral.
b) For these values of a find the exact value of $\int_0^a \frac{1}{1+x^4}dx$?
c) Let $a = \frac{1}{2}$ estimate $\int_0^a \frac{1}{1+x^4}dx$ with error less than $\frac{1}{5000}$.

6. (10 pt) Consider the function $f(x) = e^{-2x^3}$.

a) Find the Maclaurin series for this function.
b) Find $f^{(9)}(0)$.

Formula List:

(1) $$\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} = \sin(x); -\infty < x < \infty$$

(2) $$\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = \cos(x); -\infty < x < \infty$$

(3) $$\sum_{n=0}^{\infty}\frac{x^n}{n!} = e^x; -\infty < x < \infty$$

(4) $$\sum_{n=0}^{\infty}x^n = \frac{1}{1-x}; -1 < x < 1$$

(5) $$\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_{n}^{\infty} f(x)dx$$