MATH 166
SUMMER 2011
EXAM 3

1. (15 pt) Let $f(x) = \int_{\frac{\pi}{4}}^{x} \sqrt{\sec^2(t) - 2}\, dt$, $\frac{\pi}{4} \leq x < \frac{\pi}{2}$. Find the length of this curve, $\frac{\pi}{4} \leq x \leq \frac{\pi}{3}$.

2. (15 pt) A window is formed in the shape of a rectangle of vertical height a and width b with semicircles of diameter a on the left and right sides. If the top of this window is submerged D feet under the surface of a fluid with density ρ, find the force due to hydrostatic pressure on the window.

3. (60 pt) Consider a triangle in the first quadrant with vertices $(0, 0), (a, 0)$ and $(0, a)$ $(a > 0)$.
   a) Locate the $x-$coordinate of the centroid.
   b) Locate the $y-$coordinate of the centroid.
   c) Find the volume obtained when this triangle is revolved about the $x-$axis and the volume obtained when this triangle is revolved about the $y-$axis.
   d) Find the volume obtained when this triangle revolved about the line $y = -mx, m > 0$.
   e) What happens to your answer from d) as $m \longrightarrow 0^+$ and as $m \longrightarrow \infty$ (and what should happen)?
   f) For what value of m is the volume from d) maximized?

4. (20 pt) Let α be a real number such that $0 < \alpha < 1$ and consider the curve $y = \ln(x), \alpha \leq x \leq 1$.
   a) Find the surface area obtained when this curve is revolved about the $y-$axis.
   b) What happens to your answer from part a) as $\alpha \longrightarrow 0^+$?

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
(6) $\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
(7) $\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
(8) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(9) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(10) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
(11) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(12) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(13) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(14) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(15) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(16) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(17) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(18) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(19) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(20) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(21) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$