MATH 166
SUMMER 2012
EXAM 3

1. Consider the function $f(x) = \cos(ax), 0 \leq x \leq \frac{\pi}{2a}$.
   a) (15 pt) Find the surface area generated when this curve is revolved about the $x-$axis.
   b) (10 pt) What happens to your answer as $a \longrightarrow \infty$? What should happen?

2. Consider the region bounded by the semicircle $y = \sqrt{R^2 - x^2}$, and the lines $y = h$ $(h \geq R > 0)$, $x = R$, and $x = -R$.
   a) (15 pt) Locate the centroid of this region.
   b) (5 pt) What happens to your answer when $h = R$?
   c) (10 pt) For what value of h is the centroid located on the semicircle?

3. A cylindrical can of (circular) base radius R and height h is lying on its side. This can is full of a liquid of density ρ.
   a) (15 pt)Find the force due to hydrostatic pressure on one of the circular ends.
   b) (10 pt) Find the force due to hydrostatic pressure on the cylindrical side.
   c) (10 pt) For what value(s) of h, in terms of R, is the force on the cylindrical side equal to the total force on the two ends?

4. (20 pt) Consider the function $f(x) = \ln(x),\ x > 0$. Find the value(s) of a such that the length of this curve from a to $a + 1$ is minimized.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
(6) $\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
(7) $\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
(8) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(9) $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(10) $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
(11) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(12) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(13) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(14) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(15) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(16) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(17) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(18) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(19) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(20) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(21) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$