MATH 166
SUMMER 2011
EXAM 4

1. (48 pt) Determine if the following series converge or diverge.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{\sqrt[10]{3n^{26}+8n^9+9n^8+n+2}}$ b) $\displaystyle\sum_{n=0}^{\infty} \frac{(2n)!}{5^{n+1}(n!)^2}$ c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(100^n(n!)^5)}{3n^2+2}$

d) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{n\ln(n)}{n^2+1}$ e) $\displaystyle\sum_{n=3}^{\infty} \ln(\frac{n}{n+2})$ f) $\displaystyle\sum_{n=2}^{\infty} (\frac{4n+\sin(2n)}{3n})^{2n+1}$

2. (18 pt) Determine if the following sequences converge or diverge.

a) $\displaystyle(\frac{4n+\sin(2n)}{3n})_{n=1}^{\infty}$ b) $\displaystyle(n^2\tan(\frac{3}{n})\sin(\frac{5}{n}))_{n=1}^{\infty}$

c) $(\sqrt{5}, \sqrt{5+\sqrt{5}}, \sqrt{5+\sqrt{5+\sqrt{5}}}, \cdots)$

3. (24 pt) For this problem, $f(x)$ is a positive function such that $f'(x) < 0$ for all $x > 0$ and suppose that $\lim_{x\to\infty} f(x) = 0$. We let $a_n = f(n)$ for all integers $n \geq 0$.

a) Determine if the series $\sum_{n=0}^{\infty}(-1)^n a_n$ converges.
b) Show that the series $\sum_{n=0}^{\infty} a_n$ converges if and only if $\sum_{n=0}^{\infty} \sin(a_n)$ converges.
c) Show that the series $\sum_{n=0}^{\infty} a_n^n$ converges.
d) Show that the series $\sum_{n=1}^{\infty}(\frac{2}{3}+4a_n)^{2n}$ converges.

4. (10 pt) Consider the series

$$\sum_{n=1}^{\infty} \frac{2n}{(n^2+1)^2}.$$

a) Show that the hypotheses of the integral test are satified and show that this series converges. (Hint: $\frac{d}{dx}(\frac{2x}{(x^2+1)^2}) = \frac{2(1-3x^2)}{(x^2+1)^3}$.)
b) How many terms are required so that the approximation $s \approx s_n$ has error less than or equal $\frac{1}{10^6}$.

5. (10 pt) Consider the series

$$\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{n!}.$$

a) Show that this series converges.
b) Estimate the sum of this series with error less than or equal to $\frac{1}{10}$ and explain how you got your answer.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
(6) $\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
(7) $\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
(8) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(9) $\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(10) $\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
(11) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(12) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(13) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(14) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(15) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(16) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(17) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(18) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(19) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(20) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(21) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$