MATH 166
SUMMER 2012
EXAM 4

1. (48 pt) Determine if the following series converge or diverge.

$$a) \sum_{n=1}^{\infty} \frac{1}{(\ln(n))^{\ln(n)}} \qquad b) \sum_{n=0}^{\infty} (-1)^n \frac{\ln(n)}{3n+1} \qquad c) \sum_{n=0}^{\infty} \frac{(3n)!}{n!(2n)!2^{3n}}$$

$$d) \sum_{n=1}^{\infty} n \sin(\frac{1}{n^3}) \qquad e) \sum_{n=1}^{\infty} \frac{\sqrt[4]{16n^{10}+2n^5+1}}{\sqrt[5]{32n^{18}+n^8+n+7}} \qquad f) \sum_{n=2}^{\infty} (\sqrt[n]{9} - \sqrt[n+1]{9})$$

2. (20 pt) Consider the following sequence

$$a_{n+1} = \sqrt{2a_n - 1}, \; n \geq 1, \; a_1 > \frac{1}{2}.$$

a) If $a_1 \neq 1$ show that this sequence is always decreasing.
b) Show if $a_1 > 1$ show that this sequence has a floor of 1.
c) Explain why this sequence converges if $a_1 > 1$.
d) If this sequence converges, what is its limit?
e) What happens if $\frac{1}{2} < a_1 < 1$?

3. (12 pt) Determine if the following sequences converge or diverge.

$$a) \; (a, \sin(a), \sin(\sin(a)), \cdots) \qquad b) \; (\frac{(-1)^n(n^2 + n\sin(n))}{n^2+1})_{n=1}^{\infty}$$

4. (20 pt) Consider the series

$$\sum_{n=1}^{\infty} \frac{4n}{(n^2+1)^3} \text{ and } \sum_{n=1}^{\infty} (-1)^n \frac{4n}{(n^2+1)^3}$$

a) Show that the first series converges.
b) How many terms are required so that the approximation $s \approx s_n$ has error less than or equal $\frac{1}{100}$.
c) Show that the second series converges.
d) How many terms are required so that the approximation $s \approx s_n$ has error less than or equal $\frac{1}{100}$.

5. (10 pt) Consider the series

$$\sum_{n=0}^{\infty} a_n,$$

and suppose that the partial sums satify the formula

$$s_n = 3n \sin(\frac{2}{n}).$$

a) Does this series converge? If so, what is its sum?
b) What is $\lim_{n \to \infty} a_n$ or is there not enough information to tell?