MATH 166
SUMMER 2012
EXAM 6

1. Consider the parametric equations $x = t^4 - 8t^2$ and $y = 2t^3 - 3t^2$.
   a) (10 pr) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$ and the intervals of increase of x and y (a table would be appropriate for this).
   b) (6 pt) Find where this curve intersects the $x-$axis and where it intersects the $y-$axis.
   c) (9 pt) Sketch the graph of this parametric equation.
   d) (8 pt) Find the area between the $x-$axis and the part of the curve between the two points of intersection with the $x-$axis.
   e) (8 pt) Find the area bounded by the curve and the $y-$axis.

2. (24 pt) Consider the polar equation $r = 1 + 2\sin(\frac{1}{2}\theta)$.
   a) Sketch the graph of this polar equation.
   b) Find the area of the innermost loop.
   c) Find the area enclosed by this curve, excluding the (two) inner loops.

3. (24 pt) Consider the parametric equations $x = \int_0^t \sqrt{1 - |\sin(z)|}dz$ and $y = \int_0^t \sqrt{|\sin(z)|}dz$.
   a) Find the length of this curve $0 \leq t \leq a$.
   b) Find a function that gives the speed of a point moving according to these equations.
   c) Find values of t for which this curve has horizontal and vertical tangent lines.

4. (21 pt) Consider the polar equation $r = a\cos(\theta) + a\sin(\theta)$, where $a > 0$ is constant.
   a) Find the length of this curve.
   b) Find the area enclosed by this curve.
   c) Find $\frac{dy}{dx}$ and determine where this curve has horizontal and vertical tangent lines.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) Force=(pressure)(area) and pressure=ρ(depth).
(12) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2} dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$
(13) $S = \int_a^b 2\pi(x \text{ or } y) ds$
(14) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(15) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(16) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(17) $A = \int_a^b \frac{1}{2} r^2 d\theta$
(18) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(19) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$
(20) The surface area of a cone: $A = \pi r L$ where r is the radius and L is the slant height.
(21) $\frac{dy}{dx} = \frac{\frac{d}{dt}(y)}{\frac{dx}{dt}} = \frac{\frac{dr}{d\theta}\sin(\theta) + r\cos(\theta)}{\frac{dr}{d\theta}\cos(\theta) - r\sin(\theta)}$
(22) $\int \sec^5(x)\ dx = \frac{1}{4}\sec^3(x)\tan(x) + \frac{3}{8}\sec(x)\tan(x) + \frac{3}{8}\ln(|\sec(x) + \tan(x)|) + c$